Supplementary Appendix for

Growth in the Shaded Sun: The Role of International Development Finance and Corruption

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Contents

1 Appendix: Data Sources and Details

1.1 Development Finance Datasets

1.1.1 Development Assistance Committee (DAC) and non-Chinese Development Finance Data

For DAC and non-Chinese development finance data, I rely on Creditor Reporting System (CRS) datasets. CRS provides project-level dataset on development projects for each year. I manually download and append CRS datasets for 2000 through 2021 which are available on OECD website. For the analyses in the paper, I clean the data according to the following steps.

1. I keep official projects while dropping private projects. A project is classified into those categories according to the following criteria. A project is official if $flow\_name$ is either 'ODA Grants', 'ODA Loans' 'Other Official Flows (non Export Credit)'. A project is private if $flow\_name$ is "Private Development Finance".

2. To avoid double counting, an observation is dropped if $initial\_report$ code is either 2 ('revision', 6 observations), 3 ('previously reported activity (increase/decrease of earlier commitment, disbursement on earlier commitment)'), or 5 ('provisional data'). The remaining observations fall into either 1 ('new activity reported') or 8 ('commitment is estimated as equal to disbursement').

1.1.2 Chinese Development Finance Data

I rely on the AidData's Global Chinese Development Finance Dataset (version 3.0) introduced in Custer et al. (2023). It captures information on 20,985 official development projects funded by Chinese government institutions or state-owned entities between 2000 and 2021. I drop observations if $RecommendedForAggregates$ is 'No'. It is based on the pre-selected criteria by AidData. Specifically, it excludes all canceled projects, suspended projects, and projects that never reached the official commitment stage. Additionally, it avoids double counting by excluding delayed funding allocation of previously signed financial agreements and debt forgiveness activities of previous projects.

1.1.3 Consolidated Development Finance Dataset

I combine the two datasets from above to construct a consolidated dataset that encompasses both Chinese and non-Chinese development finance projects. I drop observations if recipient is an organization or a group of countries, not a country.

1.2 Corruption Perception Index

The full list of data sources used by Transparency International to construct the Corruption Perception Index is as follows.

1. African Development Bank Country Policy and Institutional Assessment
2. Bertelsmann Stiftung Sustainable Governance Indicator
3. Bertelsmann Stiftung Transformation Index
4. Economist Intelligence Unit Country Risk Service
5. Freedom House Nations in Transit
6. Global Insight Country Risk Ratings
7. IMD World Competitiveness Center World Competitiveness Yearbook Executive Opinion Survey
8. Political and Economic Risk Consultancy Asian Intelligence
9. The PRS Group International Country Risk Guide
10. World Bank Country Policy and Institutional Assessment
11. World Economic Forum Executive Opinion Survey
12. World Justice Project Rule of Law Index Expert Survey
13. Varieties of Democracy (V-Dem)

In empirical analyses, I use the index averaged over sample periods for two main reasons:

1. Methodological Change: In 2012, there was an adjustment in the CPI construction methodology, primarily involving a change in scale. This adjustment occurs within my sample period (2000-2021). To ensure comparability across the years, I normalize the pre-2012 values to match the post-2012 scaling. The average of this normalized series is used to minimize any potential bias introduced by the scale change.

2. Missing Values: Variance decomposition analysis indicates that the within-country variation in CPI is much smaller (2%) than the cross-country variation (98%) and some countries have missing annual values. Hence, using the average CPI maximizes the dataset's robustness, both temporally and cross-sectionally.

In robustness tests, I experiment with different versions of the corruption measure — the raw normalized series, average old series, and average new series — and confirm that the main results remain qualitatively unchanged.

1.3 Other Control Variables

I incorporate additional control variables from diverse sources to enrich the analysis. Macroeconomic indicators for recipient countries are sourced from the World Development Indicators (WDI). Bilateral trade data is obtained from the IMF Direction of Trade (DOT). To adjust DAC project values from current to constant dollar terms, I utilize inflator data from OECD DAC. Gravity variables, which include geographic and economic characteristics influencing trade, are drawn from the CEPII gravity database, as updated by Conte et al. (2022). Additionally, I employ Ideal Point Distance, a measure of countries' bilateral voting alignment during United Nations General Assembly sessions, constructed by Bailey et al. (2017). In robustness checks, I further include capital openness index by Chinn and Ito (2008) and Polity IV democracy index by Center for Systemic Peace.

2 Supplementary Material for Empirical Analysis

2.1 Supplementary Material for Aggregate-level Analyses

2.1.1 Panel Regression with China's share of DF inflows

Panel Regression (Country-level). Through panel regressions, I confirm that the positive correlation between China's share and recipient's corruption is not driven by specific years but consistent over time. I use OLS to estimate:

$$SHARE_{rt}^{CHN} = FE_t + \beta \cdot CORRUPT_r + \mathbf{X}_{rt} \cdot \gamma + constant + \epsilon_{rt}.$$

Here, $SHARE_{rt}^{CHN}$ represents the percentage share of the value of Chinese DF used by recipient country r in year t. Like the cross-section regression, the corruption measure, $CORRUPT_r$, is averaged over the sample period.[1] FE_t denotes time fixed effects, and $\mathbf{X}_{rt}$ includes the same control variables[2] as in the cross-country regression, measured annually instead of being averaged over the sample period.

Estimates in columns (1) and (2) of Panel (a) of Table 1 show that a standard deviation (10.9) increase in the corruption index is associated with a 6.3%p increase in the share of Chinese DF, slightly smaller than the cross-country estimate of 7.9%p. In columns (3) and (4), where the dependent variable is trimmed at 5% to exclude observations that heavily rely on either the DAC or Chinese DF, the results are qualitatively similar. The effect of a one standard deviation increase in corruption ranges from 8.6%p to 9.5%p, suggesting that the results are not driven by outliers where a recipient country relies exclusively on either Chinese or DAC DF.

Panel Regression (Sectoral Level). To examine whether the country-level results are influenced by some sector-specific characteristics potentially correlated with corruption, I conduct a sectoral-level regression incorporating sector-year fixed effects. This approach helps isolate the relationship between corruption and China's share of total DF value at the

[1] Variance decomposition shows that within-country variation accounts for only 2% of the variance in the Corruption Perception Index (CPI), justifying the use of the average CPI.

[2] *Recipient characteristics:* log initial GDP per capita in 2000, average GDP per capita growth, average log population, average external debt to GDP, average public and publicly guaranteed (PPG) debt to GDP, average net FDI inflows to GDP, average inflation, and dummies for region, oil producer, English as an official language, GATT, and WTO membership. *Recipient×donor characteristics:* average Ideal Point Distance, average bilateral trade, distance, and dummies for contiguity, legal origin, language, colonial relationship, religion, sibling, and FTA.

sectoral level. I estimate the following panel regression:

$$SHARE_{rst}^{CHN} = FE_{st} + \beta \cdot CORRUPT_r + \mathbf{X}_{rt} \cdot \gamma + constant + \epsilon_{rst}.$$

$SHARE_{rst}^{CHN}$ represents China's percentage share of the total DF value used by recipient r in sector s in year t. FE_{st} is sector×year fixed effects that absorb any sector-year-specific effects on China's share. $\mathbf{X}_{rt}$ includes the same control variables as in the country-level regression. β quantifies the correlation between corruption and China's share at the sectoral level.

The results indicate a positive correlation between corruption and reliance on Chinese DF at the sectoral level. In columns (1) and (2) of Table 1, I include all observations, while in columns (4) and (5) I trim the sample at the 5% level to exclude outliers. The findings suggest that a one standard deviation increase in corruption is associated with an approximate 1.1%p increase in China's share, varying slightly by specification. Although trimming the sample reduces its statistical significance due to a smaller sample size, the magnitude of the estimates remains consistent with the full sample. Despite the smaller effect sizes relative to the country-level estimates, these results confirm that the correlation between reliance on Chinese DF and corruption is pervasive across different sectors and not confined to a few.

Table 1: Sectoral Corruption Effect on China's Share of Total DF Inflows

Panel (a) Country-level panel regression				
	Full sample		If $SHARE_{rt}^{CHN} \in (0, 100)$	
	(1)	(2)	(3)	(4)
$CORRUPT_r$	0.575***	0.562***	0.778***	0.864***
	(0.157)	(0.154)	(0.177)	(0.182)
	(0.157)	(0.154)	(0.188)	(0.179)
Observations	1960	1960	939	939
R^2	0.184	0.234	0.219	0.247
Year FE & Recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓
Panel (b) Sectoral level panel regression				
	Full sample		If $SHARE_{rt}^{CHN} \in (0, 100)$	
	(1)	(2)	(3)	(4)
$CORRUPT_r$	0.130***	0.108**	0.093	0.137
	(0.048)	(0.041)	(0.106)	(0.090)
Observations	34548	34548	2064	2064
R^2	0.022	0.027	0.035	0.045
Sector×Year FE & Recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓

Note: Dependent variables are China's percent share in total DF inflow for each recipient-sector-year pair. Standard errors are clustered at the recipient level. Columns (1) and (2) include all observations. In columns (3) and (4), samples are restricted to observations where China's share ranges from 0 to 100 percent, ensuring inclusion of both DAC and Chinese DF.

2.1.2 Corruption and the Value of DF Inflows (Country-level)

I complement the analysis on China's share of DF inflows by also examining DF flows directly, analyzing the correlation between corruption and DAC and Chinese DF inflows separately. Using bilateral panel data, I estimate models with both Ordinary Least Squares (OLS) and Poisson Pseudo Maximum Likelihood (PPML). Section 2.1.2 presents results at the country level. Section 2.1.3 repeats the analysis at the sectoral level, with dependent variables aggregated by sector, and confirms that the main results hold consistently across sectors, indicating they are not driven by any particular sector.

The results show that the value of DAC DF inflows is negatively correlated with corruption, while that of Chinese DF inflows is positively correlated. These relationships hold at both the country and sectoral levels.

Bilateral DF flow data often contain many zeros, reflecting years in which specific donor × recipient pairs record no flows. This complicates log transformations, since zeros must otherwise be dropped. To address this, I use OLS with the log of one plus the DF value and apply PPML estimation. This follows common practice in the international trade literature, which faces similar challenges with bilateral trade data. I first describe the specifications and then present the results.

OLS. I first use OLS to investigate the correlation between corruption and bilateral DF inflows at the country level, conducting separate regressions for DAC DF and Chinese DF. For each recipient country r receiving DF from donor d in year t, I estimate:

$$\ln\left(1 + DF_{rdt}\right) = FE_{dt} + \beta \cdot \ln CORRUPT_r + \mathbf{X}_{rdt} \cdot \gamma + constant + \epsilon_{rdt} \tag{1}$$

Here, DF_{rdt} represents the total committed amounts in constant 2011 USD by donor d, for recipient r in year t. Like the regression with China's share, the corruption measure, $CORRUPT_r$, is averaged over the sample period.[3] I use the log of $CORRUPT_r$ as the main independent variable, which allows the coefficients to be interpreted as elasticities, facilitating a straightforward comparison between coefficients estimated using OLS and PPML. The log transformation does not qualitatively affect the main findings. The vector $\mathbf{X}_{rdt}$ includes the same recipient- and bilateral-level control variables as in previous regression, but measured in each year instead of being averaged over the sample periods. FE_{dt} represents donor×year fixed effects. ϵ_{rdt} is the error term. I run the regression separately for the DAC and Chinese

[3] Variance decomposition shows that within-country variation accounts for only 2% of the variance in the Corruption Perception Index (CPI), justifying the use of the average CPI. It can also alleviate potential measurement error.

projects. Note that in the regression with Chinese projects, the donor dimension becomes redundant as China is the only donor in the sample. The coefficients β from each regression reflect the elasticity of DAC and Chinese DF inflows with respect to changes in corruption, as measured by the Corruption Perception Index, and I refer to this as the "corruption effect."

PPML. As an alternative specification to address the many zero values in bilateral DF flows, I estimate the corruption effect using the Poisson Pseudo Maximum Likelihood (PPML) method following Silva and Tenreyro (2006). I estimate:

$$\mathbb{E}\left[DF_{rdt}\middle|\mathbf{X}\right] = exp\left(FE_{dt} + \beta \cdot \ln CORRUPT_r + \mathbf{X}_{rdt} \cdot \gamma + constant\right) \tag{2}$$

where $\mathbf{X}$ represents the vector of all predictor variables on the right-hand side of each equation. The exponents on the right-hand side correspond to the right-hand sides of the OLS regressions in equations (1). One advantage of PPML is that the estimated corruption effect β can be interpreted in the same way as the estimates from their OLS counterparts.

Results. Table 2 shows that the value of DAC DF inflows and that of Chinese DF inflows are negatively and positively correlated with the recipient's corruption, respectively. Columns (1) through (4) suggest that a 1% increase in the corruption measure is associated with a reduction in the DAC DF inflows by 0.86-1.52%, depending on the specifications. Note that the reduction in sample size when including recipient-donor controls is due to the unavailability of these variables for multinational donors like the World Bank and the IMF. Conversely, columns (5) through (8) indicate that Chinese DF inflows are positively correlated with corruption. Although the OLS estimates are not statistically significant, the PPML estimates are statistically significant, with all point estimates being positive, contrasting the effects seen with DAC DF. The PPML estimates indicate that a 1% increase in corruption is associated with a 2.35-3.2% increase in Chinese DF inflows.

Table 2: Effect of Corruption on the Value of Total DF Inflows

	DAC DF			
	(1)	(2)	(3)	(4)
ln $CORRUPT_r$	-1.442**	-1.524*	-0.860*	-0.871
	(0.634)	(0.781)	(0.442)	(0.568)
Observations	88,768	53,704	74,916	47,878
R^2	0.572	0.633	0.624	0.689
Specification	OLS	OLS	PPML	PPML
Donor×Year FE	✓	✓	✓	✓
Recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓
	Chinese DF			
	(5)	(6)	(7)	(8)
ln $CORRUPT_r$	4.187	4.161	2.345**	3.195***
	(3.855)	(3.947)	(1.137)	(1.059)
Observations	2,134	1,964	2,134	1,964
R^2	0.338	0.460	0.461	0.530
Specification	OLS	OLS	PPML	PPML
Donor×Year FE	✓	✓	✓	✓
Recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓

Note: Standard errors in parentheses are clustered at the recipient level. The dependent variable is the log of 1+ total DF amount for columns (1), (2), (5), and (6), and total DF amount for columns (3), (4), (7), and (8). DAC institutions are excluded from the sample for columns (2) and (4) due to the lack of recipient×donor controls. For PPML estimations, the pseudo R^2 is reported.

2.1.3 Corruption and DF Inflows (Sector-level)

Corruption and the Value of DF Inflows (Sectoral level). To confirm that the results at the country-level are not driven by certain sectors, I run OLS and PPML at the sectoral level, including sector fixed effects. The OLS specifications are:

$$\ln(1 + DF_{rdst}) = FE_{dst} + \beta_{DAC} \cdot \ln CORRUPT_r + \mathbf{X}_{rdt} \cdot \gamma_{DAC} + constant_{DAC} + \epsilon_{rdst} \quad (3)$$

$$\ln(1 + DF_{rCst}) = FE_{st} + \beta_{CHN} \cdot \ln CORRUPT_r + \mathbf{X}_{rt} \cdot \gamma_{CHN} + constant_{CHN} + \epsilon_{rCst}. \quad (4)$$

DF_{rdst} represents the value of total commitment for recipient country r by donor d in sector s for year t. FE_{dst} is donor×sector×year fixed effects, and the other predictors are the same as in the country-level regressions. The PPML counterparts are similarly defined, with the right-hand sides of the OLS specifications being the exponent of e.

Panel (b) of Table 3 shows that the country-level results are confirmed at the sectoral level, both qualitatively and quantitatively. Columns (1)-(4) indicate that the estimates of the corruption effect on the DAC DF are similar to those at the country level, both in terms of signs and magnitudes. Columns (5)-(8) report the estimates for Chinese DF. The PPML estimates are consistent with those at the country level, with values ranging from 2.28 to 3.13. Although the OLS estimates are smaller in magnitude than those at the country level, they are still positively estimated.

Corruption and the Number of DF Inflows (Sectoral level). To investigate the corruption effect on the count of DF projects, I replace the log of total DF value with the total count of DF projects by each donor in each year as the dependent variable in the country- and sectoral-level OLS regressions (Equations (1) in Section 2.1.2, and (3) and (4) in Section 2.1.3). Table 4 shows that higher corruption is significantly negatively correlated with the count of DAC projects at both the country and sectoral levels, while it is marginally positively correlated with Chinese projects. Columns (1) and (2) in Panels (a) and (b) reveal that a 1% increase in the corruption index is associated with approximately 9.4 fewer DAC projects at the country level and 0.45 fewer projects at the sectoral level. Conversely, columns (3) and (4) in Panels (a) and (b) suggest that a 1% increase in corruption leads to roughly 1.5 to 3.1 additional Chinese projects at the country level, and 0.08 to 0.15 more projects at the sectoral level, although these results lack statistical significance. Given that many Chinese projects are not reported in international statistics, and considering that more corrupt countries are less likely to transparently disclose their projects, the estimates are likely biased downward.

Table 3: Effect of Corruption on Sector-level DF Inflows

	DAC DF			
	(1)	(2)	(3)	(4)
ln $CORRUPT_r$	-1.035***	-1.024***	-0.893	-0.890
	(0.247)	(0.301)	(0.612)	(0.570)
Observations	1,495,040	1,074,080	1,028,826	788,023
R^2	0.412	0.460	0.5150	0.5726
Model	OLS	OLS	PPML	PPML
Donor×Year FE	✓	✓	✓	✓
Recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓
	Chinese DF			
	(5)	(6)	(7)	(8)
ln $CORRUPT_r$	1.013	0.733	2.267*	3.131***
	(0.612)	(0.560)	(1.223)	(1.099)
Observations	44,472	40,890	38,271	34,988
R^2	0.154	0.162	0.4597	0.4949
Model	OLS	OLS	PPML	PPML
Donor×Year FE	✓	✓	✓	✓
Recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓

Note: Standard errors in parentheses are clustered at the recipient level. The dependent variable is the log of 1+ total DF amount for columns (1) and (2), and total DF amount for columns (3) and (4). DAC institutions are excluded from the sample for columns (2) and (4) due to the lack of recipient×donor controls. For PPML estimations, the pseudo R^2 is reported.

Table 4: Corruption Effect on DF Project Count

(a) Country-level regressions				
	DAC projects		Chinese projects	
	(1)	(2)	(3)	(4)
ln $CORRUPT_{r(i)}$	-9.722***	-9.345**	3.109	1.549
	(2.515)	(4.252)	(2.132)	(1.767)
Observations	88,768	53,704	2,336	2,149
R^2	0.385	0.462	0.323	0.387
Fixed Effects	d×s×t	d×s×t	s×t	s×t
Loan dummy & recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓

(b) Sectoral level regressions				
	DAC projects		Chinese projects	
	(1)	(2)	(3)	(4)
ln $CORRUPT_{r(i)}$	-0.530***	-0.445**	0.152	0.076
	(0.144)	(0.209)	(0.105)	(0.088)
Observations	1,495,040	1,074,080	46,720	42,980
R^2	0.261	0.288	0.300	0.314
Fixed Effects	d×s×t	d×s×t	s×t	s×t
Loan dummy & recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓

Note: The dependent variables are the number of projects. Projects from DAC institutions are excluded in column (2) due to the lack of recipient by donor controls. Standard errors are clustered at the recipient level.

2.2 Supplementary Material for Project-level Analyses

2.2.1 Corruption Effect on Project Sizes by DF Providers

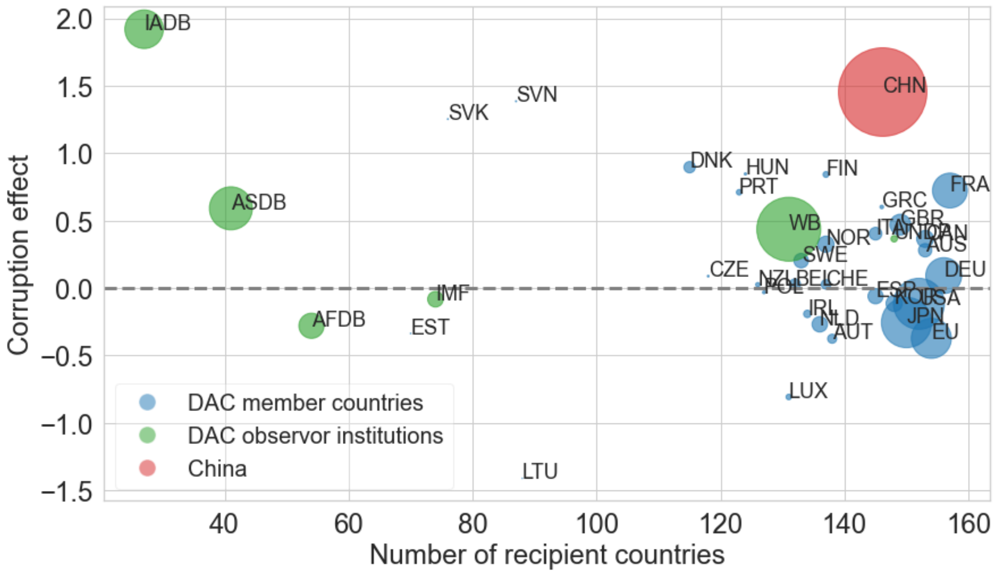


Figure 1

Note: Each circle represents a donor, with circle size proportional to the donor's total DF commitments from 2000 to 2021. Unlike this figure, the figure in the main text only includes donors that have engaged with more than 100 recipient countries for two reasons: first, the coefficients for other donors are poorly estimated due to limited observations; second, other donors are considered to play a relatively less important role as they are utilized by a smaller set of countries.

2.3 Supplementary Material for Cross-sectoral Analyses

2.3.1 Classifying Sectors by Monitoring Difficulty

I calculate the average ratings after controlling for potential confounding factors by running:

$$RATINGS_i = FE_{r(i)d(i)t(i)} + \gamma_{s(i)} + \mathbf{X}_{r(i)d(i)s(i)t(i)} \cdot \beta + constant + \epsilon_i.$$

$RATINGS_i$ represents the six-point scale rating of DF project i. $FE_{r(i)d(i)t(i)}$ denotes recipient×donor×year fixed effects, which capture both time-varying and invariant characteristics of recipient countries and donors, such as institutional quality, geography, economic or political relationships, and year-specific effects. $\mathbf{X}_{r(i)d(i)s(i)t(i)}$ includes the log of the total project amount for the recipient country in each sector, reflecting recipient-sector-specific effects related to sector size. This vector also includes dummy variables for evaluator type to control for potential biases by evaluating agencies, as well as the log of project size. The sector fixed effect, $\gamma_{s(i)}$, captures the average ratings of projects for each sector, adjusted for other effects specific to the recipient, donor, year, evaluator, project size, and sector size.

Table 5 presents the estimation results for the control variables along with the F-test results. These tests evaluate the null hypothesis that the sector fixed effects are jointly zero. The results allow me to reject this null hypothesis, with standard error clustering at various levels demonstrating that average project ratings differ significantly across sectors.

Table 5

	(1)	(2)	(3)	(4)	(5)
Log project size	0.037***	0.037**	0.037	0.037***	0.037***
	(0.012)	(0.014)	(0.024)	(0.014)	(0.015)
Log sector total DF amount	0.036**	0.036**	0.036**	0.036**	0.036**
	(0.016)	(0.017)	(0.012)	(0.016)	(0.018)
Evaluator = inde. eval. office	-0.208***	-0.208**	-0.208***	-0.208***	-0.208**
	(0.076)	(0.091)	(0.004)	(0.080)	(0.095)
Evaluator = internal	0.049	0.049	0.049	0.049	0.049
	(0.257)	(0.363)	(0.115)	(0.320)	(0.388)
Observations	8786	8786	8786	8786	8786
R^2	0.426	0.426	0.426	0.426	0.426
F (χ^2) statistic	4.81	5.88	1140.79	4.17	122.48
P-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
SE clustering	None	Recip	Donor	Recip.×Sector	Bootstrapped
Recipient×Donor×Year FE	✓	✓	✓	✓	✓
Sector dummies	✓	✓	✓	✓	✓

Note: Null hypothesis of F test is that all coefficients of the sector dummies are jointly zero.

Figure 2 depicts the OLS estimates of sector fixed effects alongside the distribution of bootstrapped estimates, illustrating the heterogeneity in average ratings across sectors. It is evident that sectors involving long-term and large-scale projects, financial transfers, or complex multi-sectoral features, such as Industry, Mining, Construction, Water Supply and Sanitation, Agriculture, Forestry, and Fishing, are ranked at the bottom with relatively small standard errors. Conversely, sectors associated with unexpected and unplanned humanitarian projects, in-kind transfers, or short-term projects, such as Emergency Response, Reconstructive Relief & Rehabilitation, Development Food Assistance, and Other Commodity Assistance, rank highly, albeit with larger standard errors. This pattern supports the conventional wisdom that managing and monitoring long-term, large-scale projects with complex structures and financial transfers is more challenging, while it is relatively easier to monitor emergency and short-term, in-kind projects. Additionally, Health and Education sectors also rank highly. This observation corroborates findings from previous literature suggesting that corrupt governments tend to reduce public expenditure on health and education, as these sectors do not offer as many lucrative opportunities for government officials compared to other sectors (Mauro, 1998).

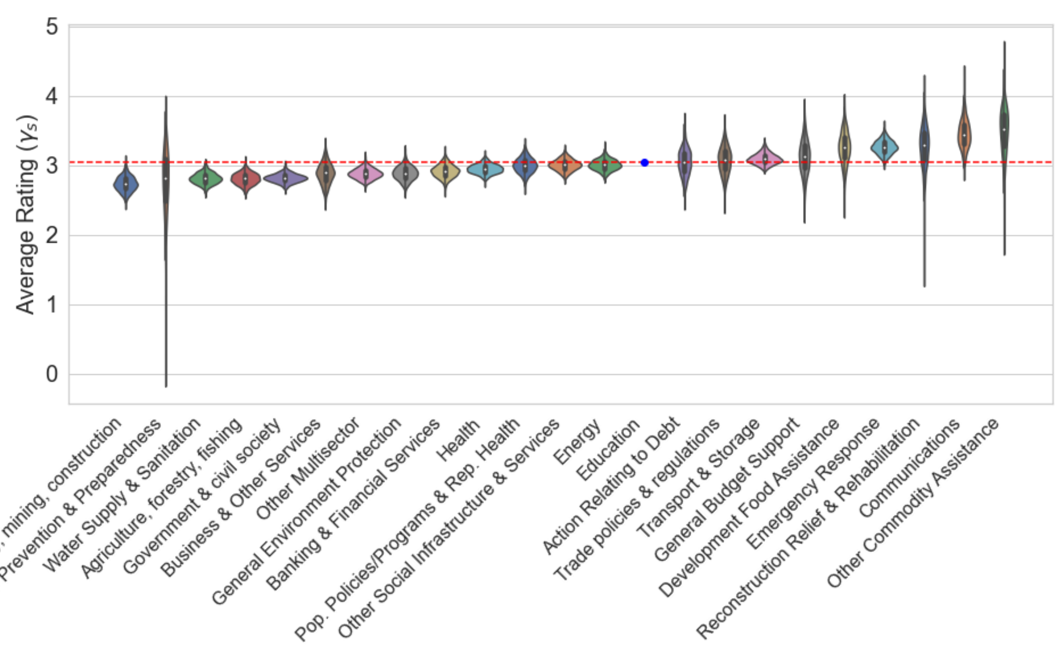


Figure 2: Bootstrapped Estimates of Sectoral Monitoring Intensity

Note: This figure shows the OLS estimate of the sector dummy coefficient from regressing DF project implementation ratings on sector dummies and other controls, along with the distribution of bootstrapped estimates for each sector dummy. The bootstrap simulation is conducted 1,000 times.

2.3.2 Corruption Effect by Corruption Quartiles

As an additional cross-sectoral exercise, I estimate the level effect and the interaction effect of corruption on project sizes across different corruption quartiles. This approach offers two significant advantages. First, it reveals whether these effects are consistently present across various corruption quartiles or if the estimates are predominantly driven by countries within specific quartiles. Second, it provides an alternative quantitative interpretation, allowing me to quantify how much larger project sizes are in countries within different corruption quartiles. This complements the elasticity interpretation used previously, which focuses on how sensitively project size responds to changes in corruption.

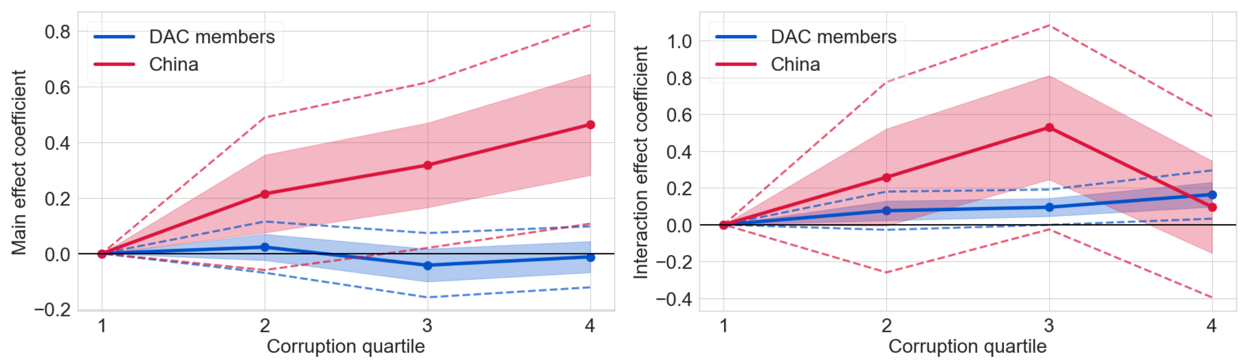
I use OLS to estimate:

$$\begin{aligned}\ln SIZE_i =& FE_{d(i)s(i)t(i)} + \sum_{q=2}^{4} \beta_q \cdot CORRUPTQ^q_{r(i)} \\ &+ \sum_{q=2}^{4} \delta_q \cdot CORRUPTQ^q_{r(i)} \times LowMonitor_{s(i)} + \mathbf{X}_{r(i)d(i)t(i)} \cdot \gamma + constant + \epsilon_i,\end{aligned}$$

where $CORRUPTQ^q_{r(i)}$ is a dummy variable that takes the value of 1 if recipient r belongs to the qth quartile with respect to the corruption measure among countries included in the previous project size regression. The other predictors are the same as in previous specifications. The coefficient β_q measures the percentage increase in project sizes for countries in the qth quartile of corruption compared to those in the least corrupt quartile (level effect). The coefficient δ_q captures the additional effect of corruption on project sizes in sectors characterized by low monitoring intensity (interaction effect). I estimate the level and interaction effects for DAC projects and Chinese projects by running the regression separately.

Figure 3 illustrates that the positive level effect of corruption on project size linearly strengthens across corruption quartiles for Chinese projects, while DAC projects exhibit no significant level effects in any quartile. The figure displays the point estimates along with 68% and 90% confidence intervals for both the level effects (β_q) and interaction effects (δ_q). Panel (a) reveals that in countries within the most corrupt quartile, project sizes are, on average, greater by 0.46%, and by 0.32% for countries in the third quartile—both statistically significant. Although the effect for the second quartile is not statistically significant at the 10% level, the estimates indicate a linear increase in the level effect of corruption on Chinese projects across all quartiles. Conversely, the level effect of corruption for DAC projects is not significantly different from zero across all quartiles, consistent with the qualitative findings from previous regressions.

Panel (b) shows that the interaction effect of monitoring difficulty and corruption on project sizes is linear across corruption quartiles for DAC DF, but nonlinear for Chinese projects. In sectors that are harder to monitor, DAC projects in the 2nd, 3rd, and 4th corruption quartiles exhibit statistically significantly larger project sizes compared to those in the least corrupt quartile, with a weak linear trend across these quartiles. However, the interaction effect for Chinese projects is statistically significant only in the third quartile, without exhibiting a linear pattern across quartiles.



(a) Level Effect

(b) Interaction Effect

Figure 3: Corruption Effect by Corruption Quartiles and Sectoral Monitoring Difficulties
Note: Each dot represents the OLS estimate of dummy variables for corruption quartiles and their interaction with binary sectoral monitoring difficulty for each donor group. Dashed lines indicate the 90% confidence intervals, and shaded areas represent the 68% confidence intervals. Standard errors are clustered at the recipient level. Table 6 reports the estimates and regression statistics.

Table 6

	(1)	(2)	(3)	(4)
(a) DAC projects				
$CORRUPT_{r(i)}$ Q4	0.076	0.046	0.021	-0.012
	(0.058)	(0.053)	(0.056)	(0.056)
$CORRUPT_{r(i)}$ Q3	0.012	-0.011	-0.015	-0.042
	(0.050)	(0.055)	(0.053)	(0.059)
$CORRUPT_{r(i)}$ Q2	0.054	0.048	0.040	0.023
	(0.042)	(0.044)	(0.045)	(0.047)
$CORRUPT_{r(i)}$ Q4 $\times$ $LowMonitor_{s(i)}$			0.155***	0.164**
			(0.058)	(0.067)
$CORRUPT_{r(i)}$ Q3 $\times$ $LowMonitor_{s(i)}$			0.082*	0.095*
			(0.042)	(0.049)
$CORRUPT_{r(i)}$ Q2 $\times$ $LowMonitor_{s(i)}$			0.044	0.076
			(0.045)	(0.053)
Observations	1,183,235	1,045,455	1,155,291	1,021,935
R^2	0.354	0.265	0.355	0.264
(b) Chinese projects				
$CORRUPT_{r(i)}$ Q4	0.300*	0.489**	0.272	0.464**
	(0.169)	(0.188)	(0.170)	(0.182)
$CORRUPT_{r(i)}$ Q3	0.467***	0.458***	0.340*	0.318**
	(0.174)	(0.158)	(0.182)	(0.152)
$CORRUPT_{r(i)}$ Q2	0.244	0.302*	0.160	0.215
	(0.151)	(0.156)	(0.144)	(0.140)
$CORRUPT_{r(i)}$ Q4 $\times$ $LowMonitor_{s(i)}$			0.107	0.097
			(0.249)	(0.251)
$CORRUPT_{r(i)}$ Q3 $\times$ $LowMonitor_{s(i)}$			0.468	0.529*
			(0.289)	(0.283)
$CORRUPT_{r(i)}$ Q2 $\times$ $LowMonitor_{s(i)}$			0.250	0.258
			(0.259)	(0.264)
Observations	7,559	7,559	7,439	7,439
R^2	0.658	0.662	0.658	0.663
Donor×Sector×Year FE	✓	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓	✓
Other recipient controls	✓	✓	✓	✓
Recipient×Donor controls		✓		✓
SE clustering	Recipient	Recipient	Recipient	Recipient

2.4 Additional Robustness Checks and Exercises

2.4.1 2SLS with Settler Mortality as an Instrument.

There is a possibility that the Corruption Perception Index (CPI) used in the main analysis might be correlated with some omitted variables. To check the robustness of the main findings, I employ an instrumental variable approach. Following Acemoglu et al. (2001), I use settler mortality in recipient countries during the colonial era as an instrument for corruption. This exercise qualitatively confirms the baseline results that recipient corruption is positively correlated with Chinese project size, an effect not observed for DAC projects.

This approach exploits institutional differences among countries colonized by Europeans and is based on three premises. First, different types of colonization strategies were employed. In some colonies, Europeans set up extractive institutions that provided little protection for private property and few checks against government diversion. The primary purpose of these institutions was to transfer resources from the colonies to the colonizers. In other colonies, Europeans migrated and settled, replicating European institutions with strong private property protection and checks against government diversion. The second premise is that these colonization strategies were largely influenced by the feasibility of settlement, which was mainly determined by the disease environment. The third premise is that colonial institutions persist even after independence, with extractive institutions continuing to serve as diversion tools for the local government instead of the colonizers.

Based on these premises, Acemoglu et al. (2001) use data on the mortality rates of soldiers, bishops, and sailors stationed in the colonies between the seventeenth and nineteenth centuries as an instrument for current institutional quality. In a similar vein, I use the mortality rate as an instrument for the current Corruption Perception Index.

The second-stage regression is the same as in the main text. In the first stage, I run the following regression:[4]

$$\ln CPI_{r(i)} = FE_{d(i)s(i)t(i)} + \beta \cdot \ln Mortality_{r(i)} + \mathbf{X}_{r(i)d(i)t(i)} \cdot \gamma + constant + \nu_i.$$

The first-stage regression includes all the fixed effects and control variables used in the second stage. The results are summarized in Table **??**. The first-stage results in panel (c) show that higher settler mortality predicts lower CPI, equivalently higher corruption, which is consistent with the theory. The Cragg-Donald Wald F-statistic indicates that the instrument is strong if the error terms are independent. However, the Kleibergen-Paap rk Wald F-statistic and rk LM p-value suggest some possibility of a weak instrument if the

[4] The dependent variable and mortality rate vectors are a stack of repeated recipient-specific values over different combinations of donor, sector, and time.

error terms are not independent. Consequently, the second-stage coefficients for log CPI are not very precisely estimated. Nonetheless, the point estimates are consistent with the main exercises: the estimate for DAC members is close to zero, while the estimate for China is positive and of much greater magnitude. It is important to note that many observations are dropped compared to the baseline analysis, as settler mortality data is only available for countries that had been colonized by Europeans.

Table 7

	DAC members		China	
	(1) OLS	(2) IV	(3) OLS	(4) IV
(a) OLS				
$CORRUPT_{r(i)}$	0.006		0.022*	
	(0.004)		(0.012)	
(b) IV Second-stage				
$CORRUPT_{r(i)}$		-0.011		0.036
		(0.012)		(0.026)
(c) IV First-stage				
$Mortality_{r(i)}$		1.846**		2.104***
		(0.709)		(0.770)
Observations	747,357	747,357	5,005	5,005
R^2 (first-stage R^2 for IV)	0.269	0.421	0.688	0.576
Cragg-Donald Wald F stat.		4.9e+04		464.141
Kleibergen-Paap rk Wald F stat.		6.790		7.467
Kleibergen-Paap rk LM (P-value)		0.0388		0.0339
Donor×Sector×Year FE	✓	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓	✓
Other recipient controls	✓	✓	✓	✓
Recipient×Donor controls	✓	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient	Recipient

2.4.2 Outlier Treatments.

In the baseline analysis of project size, I include all observations of projects with a positive commitment amount. To test the robustness of the main results and explore whether they are influenced by outliers, I vary the treatment of outliers. Table 8 reports the estimated corruption effect when outliers are winsorized at 1%, at 2%, and trimmed at 1% and 2%. The results are not qualitatively different.

Table 8

	Baseline	Winsor (1%)	Winsor (2%)	Trim (1%)	Trim (2%)
	(1)	(2)	(3)	(4)	(5)
(a) DAC member countries					
$CORRUPT_{r(i)}$	-0.023	-0.014	-0.012	-0.015	-0.018
	(0.129)	(0.128)	(0.125)	(0.118)	(0.111)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.353**	0.345**	0.343**	0.329**	0.317**
	(0.164)	(0.157)	(0.152)	(0.140)	(0.121)
Observations	1,021,935	1,021,935	1,021,935	1,001,389	980,976
R^2	0.264	0.259	0.256	0.235	0.220
(b) projects by China					
$CORRUPT_{r(i)}$	1.376***	1.345***	1.332***	1.325***	1.182***
	(0.459)	(0.451)	(0.445)	(0.425)	(0.403)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.449	0.447	0.403	0.269	-0.097
	(0.708)	(0.692)	(0.669)	(0.598)	(0.515)
Observations	7,439	7,439	7,439	7,291	7,151
R^2	0.662	0.666	0.669	0.662	0.660
Donor×Sector×Year FE	✓	✓	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓	✓	✓
Other recipient controls	✓	✓	✓	✓	✓
Recipient×Donor controls	✓	✓	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient	Recipient	Recipient

2.4.3 Alternative Corruption Measure.

In the main analysis of project size, I use the average Corruption Perception Index (CPI) of recipient countries over the sample period. To confirm the robustness, I use the raw normalized CPI over 2000-2021, the old CPI averaged over 2000-2011, and the new CPI averaged over 2012-2021. Table 9 shows that the estimates are qualitatively similar to the baseline results.

Table 9

	Baseline	Normalized CPI	Avg. old CPI (0-10)	Old CPI (0-10)
	(1)	(2)	(3)	(4)
(a) DAC projects				
$CORRUPT_{r(i)}$	-0.023	-0.091	-0.891	-2.636
	(0.129)	(0.123)	(2.567)	(2.401)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.353**	0.308*	6.305**	6.988**
	(0.164)	(0.170)	(2.814)	(2.703)
Observations	1,021,935	987,837	1,021,935	412,323
R^2	0.264	0.262	0.264	0.254
(b) Chinese projects				
$CORRUPT_{r(i)}$	1.376***	1.257***	27.034***	21.696***
	(0.459)	(0.448)	(6.763)	(7.544)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.449	0.445	12.418	-4.602
	(0.708)	(0.893)	(14.647)	(11.583)
Observations	7,439	7,030	7,439	2,175
R^2	0.662	0.666	0.663	0.635
Donor×Sector×Year FE	✓	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓	✓
Other recipient controls	✓	✓	✓	✓
Recipient×Donor controls	✓	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient	Recipient

2.4.4 Alternative Monitoring Difficulty Measure.

In the main text, I use a binary version of sectoral monitoring difficulty for straightforward interpretation. Table 10 confirms that the baseline findings are qualitatively robust to alternative monitoring intensity measures, including a continuous one.

Table 10

	Binary (=1 if ≤ Q1)	Binary (=1 if ≤ Q2)	Continuous (-1× Monitor)
	(1)	(2)	(3)
(a) DAC projects			
$CORRUPT_{r(i)}$	-0.023	-0.190	-0.067
	(0.129)	(0.135)	(0.122)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.353**	0.008***	0.025***
	(0.164)	(0.002)	(0.009)
Observations	1,021,935	1,021,935	1,021,935
R^2	0.264	0.264	0.264
(b) Chinese projects			
$CORRUPT_{r(i)}$	1.376***	1.196**	1.383***
	(0.459)	(0.477)	(0.466)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.449	0.009	0.026
	(0.708)	(0.008)	(0.021)
Observations	7,439	7,439	7,439
R^2	0.662	0.662	0.662
Donor×Sector×Year FE	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓
Other recipient controls	✓	✓	✓
Recipient×Donor controls	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient

2.4.5 Placebo Test.

The significant estimates of the interaction between corruption and sectoral monitoring difficulty for DAC projects might be capturing the interaction effects of sectoral monitoring difficulty with other recipient characteristics correlated with corruption. To address this possibility, I conduct a placebo test that includes various interactions between other control variables and sectoral monitoring intensity. Table 11 shows that in all specifications, the interaction effect of corruption is significantly positive. Table 12 reports the coefficients of all placebo interaction terms.

Table 11

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$CORRUPT_{r(i)}$	-0.023	-0.020	0.008	-0.012	-0.017	-0.003	0.023
	(0.129)	(0.129)	(0.137)	(0.132)	(0.136)	(0.135)	(0.138)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.353**	0.347**	0.259*	0.325**	0.330**	0.290**	0.218**
	(0.164)	(0.156)	(0.132)	(0.140)	(0.128)	(0.121)	(0.109)
Observations	1,021,935	1,021,935	1,021,935	1,021,935	1,021,935	1,021,935	1,021,935
R^2	0.264	0.264	0.265	0.265	0.265	0.265	0.265
Recipient region × $LowMonitor_{s(i)}$		✓				✓	✓
Population / GDP PC × $LowMonitor_{s(i)}$			✓		✓		✓
Recipient character. × $LowMonitor_{s(i)}$			✓				✓
Recipient×Donor character. × $LowMonitor_{s(i)}$				✓			✓
All continuous controls × $LowMonitor_{s(i)}$					✓		✓
All dummy controls × $LowMonitor_{s(i)}$						✓	✓
Donor×Sector×Year FE	✓	✓	✓	✓	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓	✓	✓	✓	✓
Other recipient controls	✓	✓	✓	✓	✓	✓	✓
Recipient×Donor controls	✓	✓	✓	✓	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient	Recipient	Recipient	Recipient	Recipient

Table 12

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$CORRUPT_{r(i)}$	-0.023	-0.020	0.008	-0.012	-0.017	-0.003	0.023
	(0.129)	(0.129)	(0.137)	(0.132)	(0.136)	(0.135)	(0.138)
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.353**	0.347**	0.259*	0.325**	0.330**	0.290**	0.218**
	(0.164)	(0.156)	(0.132)	(0.140)	(0.128)	(0.121)	(0.109)
America × $LowMonitor_{s(i)}$		0.029				0.074	0.119*
		(0.051)				(0.065)	(0.065)
Asia × $LowMonitor_{s(i)}$		0.076				0.053	0.125***
		(0.048)				(0.048)	(0.044)
Middle East × $LowMonitor_{s(i)}$		0.119				0.042	0.083
		(0.072)				(0.096)	(0.095)
Oceania × $LowMonitor_{s(i)}$		0.085				0.088	-0.024
		(0.069)				(0.064)	(0.075)
Europe × $LowMonitor_{s(i)}$		-0.001				-0.078	-0.076
		(0.108)				(0.110)	(0.102)
GDP PC growth × $LowMonitor_{s(i)}$			-0.396		-0.158		-0.594
			(0.355)		(0.333)		(0.371)
Inflatioin × $LowMonitor_{s(i)}$			-0.000		-0.000		0.000
			(0.000)		(0.000)		(0.000)
Public debt/GDP × $LowMonitor_{s(i)}$			-0.000		-0.000		-0.000
			(0.001)		(0.001)		(0.001)
FDI inflows/GDP × $LowMonitor_{s(i)}$			-0.000		0.000		-0.000
			(0.002)		(0.002)		(0.002)
Oil producer × $LowMonitor_{s(i)}$			0.000			-0.102***	-0.002
			(0.038)			(0.035)	(0.034)
English × $LowMonitor_{s(i)}$			-0.033			0.030	0.035
			(0.047)			(0.046)	(0.044)
GATT × $LowMonitor_{s(i)}$			-0.062			-0.055	-0.016
			(0.042)			(0.055)	(0.049)
WTO × $LowMonitor_{s(i)}$			-0.058			-0.066	-0.099*
			(0.056)			(0.061)	(0.054)
Log population × $LowMonitor_{s(i)}$			-0.037***		-0.043***		-0.050***
			(0.013)		(0.010)		(0.011)
Log GDP PC × $LowMonitor_{s(i)}$			-0.042		-0.027		-0.054**
			(0.026)		(0.023)		(0.024)
Contiguous × $LowMonitor_{s(i)}$				-0.091		-0.120	0.011
				(0.135)		(0.108)	(0.153)
Common leg. origin (pre) × $LowMonitor_{s(i)}$				-0.023		-0.016	-0.017
				(0.083)		(0.074)	(0.054)
Common leg. origin (post) × $LowMonitor_{s(i)}$				-0.047		-0.040	-0.047
				(0.067)		(0.064)	(0.040)
Common language × $LowMonitor_{s(i)}$				-0.141**		-0.135**	-0.137**
				(0.059)		(0.062)	(0.057)
Common colonizer × $LowMonitor_{s(i)}$				0.195		0.284	0.371
				(0.270)		(0.274)	(0.260)
Distance × $LowMonitor_{s(i)}$				-0.000	0.000		0.000
				(0.000)	(0.000)		(0.000)
Common religion × $LowMonitor_{s(i)}$				-0.067		-0.104	-0.125
				(0.082)		(0.082)	(0.080)
Sibling ever × $LowMonitor_{s(i)}$				-0.037		-0.067	-0.037
				(0.075)		(0.080)	(0.079)
Colony ever × $LowMonitor_{s(i)}$				0.184***		0.160**	0.151**
				(0.070)		(0.065)	(0.063)
Ideal Point Distance × $LowMonitor_{s(i)}$				-0.012	0.003		-0.037
				(0.034)	(0.033)		(0.030)
Bilateral trade × $LowMonitor_{s(i)}$				-0.004	-0.006		-0.005
				(0.004)	(0.004)		(0.004)
FTA × $LowMonitor_{s(i)}$				-0.074			-0.024
				(0.048)			(0.042)
Observations	1,021,935	1,021,935	1,021,935	1,021,935	1,021,935	1,021,935	1,021,935
R^2	0.264	0.264	0.265	0.265	0.265	0.265	0.265

2.4.6 Direct Measure of Diversion Risk.

I replace the Corruption Perception Index (CPI) with indices that more directly measure the public sector diversion risk in recipient countries. While the CPI is a holistic measure of public sector corruption, it may capture aspects not directly relevant to diversion. To ensure that diversion motives play a significant role, I use the Public Corruption Index and the Executive Corruption Index from V-Democracy. These indices specifically measure the prevalence of expropriation and bribery in the public sector and among executives, respectively. I repeat the interaction regression with these alternative measures and confirm the baseline results. Table 13 reports these results.

Table 13

	Baseline (CPI)	Public misapp.	Executive misapp.
	(1)	(2)	(3)
(a) DAC projects			
$CORRUPT_{r(i)}$	-0.023		
	(0.129)		
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.353**		
	(0.164)		
Public diversion index		-0.010	
		(0.035)	
Public misapp. $\times$ $LowMonitor_{s(i)}$		0.015	
		(0.040)	
Executive diversion index			-0.019
			(0.031)
Executive misapp. $\times$ $LowMonitor_{s(i)}$			0.011
			(0.033)
Observations	1,021,935	1,020,106	1,020,106
R^2	0.264	0.264	0.264
(b) Chinese projects			
$CORRUPT_{r(i)}$	1.376***		
	(0.459)		
$CORRUPT_{r(i)} \times LowMonitor_{s(i)}$	0.449		
	(0.708)		
Public diversion index		0.335***	
		(0.105)	
Public misapp. $\times$ $LowMonitor_{s(i)}$		-0.100	
		(0.236)	
Executive diversion index			0.225**
			(0.092)
Executive misapp. $\times$ $LowMonitor_{s(i)}$			-0.260
			(0.178)
Observations	7,439	7,333	7,333
R^2	0.662	0.665	0.665
Donor×Sector×Year FE	✓	✓	✓
Loan dummy, Population, GDP PC	✓	✓	✓
Other recipient controls	✓	✓	✓
Recipient×Donor controls	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient

Note: Both measures are continuous between zero and one, with higher values indicating higher corruption, unlike the CPI.

2.4.7 Public Capital Stock, Capital Openness, and Democracy

I test the robustness of the baseline results at the project, sectoral, and country levels by including additional control variables. These variables were not used in the main analyses due to their limited availability across a significant number of countries or years. I include the log of the total public capital stock to control for potential differential effects by the relative size of the public sector, the capital openness index from Chinn and Ito (2008) to account for the effect of recipient countries' capital control policies on DF flows, and the Polity IV score to control for the impact of the degree of democracy on DF flows. Table 14 reports the results, indicating that the main findings are qualitatively unaffected.

Table 14

	(1)	(2)	(3)	(4)
	Log project size	$SHARE^{CHN}_{rst}$	Total amount	$SHARE^{CHN}_{rt}$
(a) DAC DF				
$CORRUPT_r$	-0.104***		-1.760***	
	(0.125)		(0.417)	
$CORRUPT_r \times LowMonitor_s$	0.333**			
	(0.158)			
Observations	754334		34387	
R^2	0.261		0.706	
(b) Chinese DF				
$CORRUPT_r$	1.610***	0.545***	3.268***	0.645***
	(0.606)	(0.146)	(1.195)	(0.201)
$CORRUPT_r \times LowMonitor_s$	0.663	-0.093***		
	(0.790)	(0.027)		
Observations	4,811	2,954	1,395	1,082
R^2	0.623	0.101	0.595	0.319
Level	Project	Sector	Country	Country
Model	OLS	OLS	PPML	OLS
Fixed Effects	Donor×Sector×Year	Sector×Year	Donor×Year	Year
Recipient controls	✓	✓	✓	✓
Recipient×Donor controls	✓	✓	✓	✓
Capital openness (Chinn-Ito)	✓	✓	✓	✓
Democracy (Polity IV)	✓	✓	✓	✓
Log public capital	✓	✓	✓	✓
SE clustering	Recipient	Recipient	Recipient	Recipient

3 Omitted Proofs and Extensions

3.1 Proof of Lemma 1

Let's set a Lagrangian for the government's planning problem.

$$
\begin{aligned}
\mathcal{L} = &\sum_{t=0}^{\infty} \beta^t \tilde{U}(C_t, G_t^X) \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_t \Big(Y_t + (1-\delta_K)K_t + \sum_{s\in S}\int_{j\in J_s} (1-\delta_s^E) g_{s,j,t}^E dj - C_t - K_{t+1} - \sum_{s\in S}\int_{j\in J_s} (R_s^D d_{s,j,t}^D + R_s^C d_{s,j,t}^C + \mathbb{I}_{s,j,t}^D f_s^D + \mathbb{I}_{s,j,t}^C f_s^C) dj \Big) \\
&+ \sum_{t=0}^{\infty}\sum_{s\in S}\int_{j\in J_s} \beta^{t+1}\mu_{s,j,t+1}^E \Big(g_{s,j,t+1}^E - \psi_s^D d_{s,j,t+1}^D - \psi_s^C d_{s,j,t+1}^C \Big) dj + \sum_{t=0}^{\infty}\sum_{s\in S}\int_{j\in J_s} \beta^{t+1}\mu_{s,j,t+1}^X \Big(d_{s,j,t+1}^D + d_{s,j,t+1}^C - g_{s,j,t+1}^E \Big) dj \\
&+ \sum_{t=0}^{\infty}\sum_{s\in S}\int_{j\in J_s} \beta^{t+1}\mu_{s,j,t+1}^D d_{s,j,t+1}^D dj + \sum_{t=0}^{\infty}\sum_{s\in S}\int_{j\in J_s} \beta^{t+1}\mu_{s,j,t+1}^C d_{s,j,t+1}^C dj
\end{aligned}
$$

Then, the first order condition for C_{t+1} is

$$[C_{t+1}]: \quad \tilde{U}_C'(C_{t+1}, G_{t+1}^X) = \lambda_{t+1}.$$

The first order conditions for $d_{s,j,t+1}^D$ and $d_{s,j,t+1}^C$ are

$$
\begin{aligned}
[d_{s,j,t+1}^D]: &\quad \tilde{U}_{G^X}'(C_{t+1}, G_{t+1}^X) - \lambda_{t+1}R_s^D - \mu_{s,j,t+1}^E \psi_s^D + \mu_{s,j,t+1}^X + \mu_{s,j,t+1}^D = 0 \\
[d_{s,j,t+1}^C]: &\quad \tilde{U}_{G^X}'(C_{t+1}, G_{t+1}^X) - \lambda_{t+1}R_s^C - \mu_{s,j,t+1}^E \psi_s^C + \mu_{s,j,t+1}^X + \mu_{s,j,t+1}^C = 0.
\end{aligned}
$$

GHH preference implies that $\tilde{U}_{G^X}'/\tilde{U}_C' = \chi$. Substituting for λ_{t+1} and using the GHH assumption, $[d_{s,j,t+1}^D]$ and $[d_{s,j,t+1}^C]$ can be rearranged as

$$
\begin{aligned}
[d_{s,j,t+1}^D]: &\quad \chi - R_s^D - \psi_s^D \frac{\mu_{s,j,t+1}^E}{\lambda_{t+1}} + \frac{\mu_{s,j,t+1}^X}{\lambda_{t+1}} + \frac{\mu_{s,j,t+1}^D}{\lambda_{t+1}} = 0 \\
[d_{s,j,t+1}^C]: &\quad \chi - R_s^C - \psi_s^C \frac{\mu_{s,j,t+1}^E}{\lambda_{t+1}} + \frac{\mu_{s,j,t+1}^X}{\lambda_{t+1}} + \frac{\mu_{s,j,t+1}^C}{\lambda_{t+1}} = 0.
\end{aligned}
$$

I prove by contradiction that it is not optimal to use both DF sources for project j. Suppose that both DF are used so that $d_{s,j,t+1}^D > 0$ and $d_{s,j,t+1}^C > 0$. By complementary slackness, $\mu_{s,j,t+1}^D = \mu_{s,j,t+1}^C = 0$. Note that either the monitoring constraint or the non-negativity constraint for diversion should be slack by construction. In other words, $\mu_{s,j,t+1}^E = 0$ or $\mu_{s,j,t+1}^C = 0$. I show that in either case, it is contradictory that both DF are used. First, suppose $\mu_{s,j,t+1}^E = 0$. Then, $[d_{s,j,t+1}^D]$ implies that $\frac{\mu_{s,j,t+1}^X}{\lambda_{t+1}} = R_s^D - \chi$ while $[d_{s,j,t+1}^D]$ implies that $\frac{\mu_{s,j,t+1}^X}{\lambda_{t+1}} = R_s^C - \chi$. Since $R_s^C \neq R_s^D$, it is contradictory. Now suppose that $\mu_{s,j,t+1}^C = 0$. Similarly, $[d_{s,j,t+1}^D]$ and $[d_{s,j,t+1}^C]$ can be satisfied at the same time only in a knife-edge case where $(\chi - R_s^D)/\psi_s^D = (\chi - R_s^C)/\psi_s^C$. Hence, the government finances each project j with only one DF source. $\square$

3.2 Proof of Lemma 2

Consider the Lagrangian for the government's planning problem as in the proof of Lemma 1. By Lemma 1, project j is financed by only one DF source. Suppose it is financed by $p \in \{D, C\}$. With the GHH preference assumption, the first order condition for $d^p_{s,j,t+1}$ can be modified as

$$[d^D_{s,j,t+1}]: \quad \chi - R^p_s - \psi^p_s \frac{\mu^E_{s,j,t+1}}{\lambda_{t+1}} + \frac{\mu^X_{s,j,t+1}}{\lambda_{t+1}} + \frac{\mu^p_{s,j,t+1}}{\lambda_{t+1}} = 0.$$

Since $d^p_{s,j,t+1} > 0$, complementary slackness implies that $\mu^p_{s,j,t+1} = 0$. Meanwhile, it is impossible by construction that the monitoring constraint and the non-negativity constraint for diversion bind at the same time. Hence, either $\mu^E_{s,j,t+1} = 0$ or $\mu^X_{s,j,t+1} = 0$ should hold. Suppose that $\mu^E_{s,j,t+1} = 0$. Then, $\frac{\mu^X_{s,j,t+1}}{\lambda_{t+1}} = R^p_s - \chi$. Since $\frac{\mu^X_{s,j,t+1}}{\lambda_{t+1}} \geq 0$, this is possible only if $R^p_s \geq \chi$. Moreover, if $R^p_s > \chi$, $\mu^X_{s,j,t+1} > 0$ and the non-negativity constraint for diversion should bind resulting in $g^E_{s,j,t+1} = d^p_{s,j,t+1}$. Now, suppose that $\mu^X_{s,j,t+1} = 0$. Then, $\frac{\mu^E_{s,j,t+1}}{\lambda_{t+1}} = (\chi - R^p_s)/\psi^p_s$. Since $\frac{\mu^E_{s,j,t+1}}{\lambda_{t+1}} \geq 0$, this is possible only if $R^p_s \leq \chi$. Moreover, if $R^p_s < \chi$, $\mu^E_{s,j,t+1} > 0$ and the monitoring constraint should bind resulting in $g^E_{s,j,t+1} = \psi^p_s d^p_{s,j,t+1}$. Since $R^p_s < \chi$ and $R^p_s > \chi$ are mutually exclusive and collectively exhaustive except for the knife-case where $R^p_s = \chi$, it concludes the proof. $\square$

3.3 Proofs of Lemma 3 and Corollary 1

Consider the Lagrangian $\mathcal{L}$ for the government's planning problem. Lemma 1 implies that each project j is financed by one DF source. Suppose it is financed by $p \in \{D, C\}$. The FOCs for the effective public capital in project j, $g^E_{s,j,t+1}$, and the p debt stock for j, $d^p_{s,j,t+1}$, can be rearranged as

$$[g^E_{s,j,t+1}]: \quad -\chi + mpg^E_{s,j,t+1} + 1 - \delta^E_s + \frac{\mu^E_{s,j,t+1}}{\lambda_{t+1}} - \frac{\mu^X_{s,j,t+1}}{\lambda_{t+1}} = 0$$

$$[d^D_{s,j,t+1}]: \quad \chi - R^p_s - \psi^p_s \frac{\mu^E_{s,j,t+1}}{\lambda_{t+1}} + \frac{\mu^X_{s,j,t+1}}{\lambda_{t+1}} = 0$$

where $mpg^E_{s,j,t+1} \equiv \frac{\partial Y_{t+1}}{\partial g^E_{s,j,t+1}}$. If $\chi < R^p_s$, by Lemma 2, the government chooses zero diversion hence $\mu^E_{s,j,t+1} = 0$ and $\mu^X_{s,j,t+1}/\lambda_{t+1} = R^p_s - \chi$. Plugging these into $[g^E_{s,j,t+1}]$,

$$mpg^E_{s,j,t+1} + 1 - \delta^E_s = R^p_s.$$

Now suppose $\chi > R^p_s$. Lemma 2 implies that the government chooses maximal diversion hence $\mu^X_{s,j,t+1} = 0$ and $\mu^E_{s,j,t+1}/\lambda_{t+1} = (\chi - R^p_s)/\psi^p_s$. Plugging theses into $[g^E_{s,j,t+1}]$ yields

$$\psi^p_s(mpg^E_{s,j,t+1} + 1 - \delta^E_s) + (1 - \psi^p_s)\chi = R^p_s.$$

It concludes the proof of Lemma 3. Corollary 1 can be proven simply by rearranging the last two equations so that only $mpg^E_{s,j,t+1}$ remains on the left hand side. □

3.4 Proof of Proposition 1

Lemma 1 and 2 imply that for each project, the government chooses among 4 financing options (2 by 2); DAC versus China and maximal versus zero diversion. Lemma 3 pins down the optimal size of a project when financed with each of the 4 options as $\bar{g}^{Ep}_{s,j,t}$ such that $mpg^E_{s,j,t} = \tilde{R}^p_s$. If a project is financed without diversion, the contribution of the project to the utility of the government is

$$\tilde{U}'_C \cdot \left(\int_0^{\bar{g}^{Ep}_{s,j,t}} (mpg^E_{s,j,t} + (1 - \delta^E_s) - R^p_s) dg - f^p_s \right).$$

With maximal diversion, it is

$$\begin{aligned}
&\tilde{U}'_C \cdot \left(\int_0^{\bar{g}^{Ep}_{s,j,t}} (mpg^E_{s,j,t} + (1 - \delta^E_s) - \frac{R^p_s}{\psi^p_s}) dg - f^p_s \right) + \tilde{U}'_{G^X} \cdot \left(\frac{1 - \psi^p_s}{\psi^p_s} \bar{g}^{Ep}_{s,j,t} \right) \\
&= \tilde{U}'_C \cdot \left[\int_0^{\bar{g}^{Ep}_{s,j,t}} (mpg^E_{s,j,t} + (1 - \delta^E_s) - \frac{R^p_s}{\psi^p_s}) dg - f^p_s + \frac{\tilde{U}'_{G^X}}{\tilde{U}'_C} \frac{1 - \psi^p_s}{\psi^p_s} \bar{g}^{Ep}_{s,j,t} \right] \\
&= \tilde{U}'_C \cdot \left[\int_0^{\bar{g}^{Ep}_{s,j,t}} (mpg^E_{s,j,t} + (1 - \delta^E_s) - \frac{R^p_s}{\psi^p_s} + \frac{1 - \psi^p_s}{\psi^p_s} \chi) dg - f^p_s \right] \\
&= \tilde{U}'_C \cdot \left[\int_0^{\bar{g}^{Ep}_{s,j,t}} (mpg^E_{s,j,t} + (1 - \delta^E_s) - \frac{R^p_s - (1 - \psi^p_s)\chi}{\psi^p_s}) dg - f^p_s \right]
\end{aligned}$$

Using the definition of $\tilde{R}^p_s$ and $\tilde{\pi}^p_{s,j,t}$, either without diversion or with maximal diversion, the contribution of the project to the government's utility can be written as $\tilde{U}'_C \cdot \tilde{\pi}^p_{s,j,t}$. Note that the choice of financing options affects the Lagrangian for the planning problem only through this term. Since $\tilde{U}'_C$ is a common factor, it is optimal for the government to choose the financing option that maximizes the effective profit $\tilde{\pi}^p_{s,j,t}$. □

3.5 Proof of Lemma 4

In an optimal allocation, the cutoffs can be expressed in terms of output Y_t and the effective public capital stock in sector s for period t, $G^E_{s,t}$, as follows:

$$\bar{\theta}^p_{s,t} = \frac{\left((\sigma_s-1)f^p_s\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^p_s)^{\frac{\sigma_s-1}{\sigma_s}},$$
$$\bar{\theta}^I_{s,t} = \frac{\left((\sigma_s-1)(f^p_s-f^{p'}_s)\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^p_s\tilde{R}^{p'}_s)^{\frac{\sigma_s-1}{\sigma_s}}\left[\frac{1}{(\tilde{R}^{p'}_s)^{\sigma_s-1}-(\tilde{R}^p_s)^{\sigma_s-1}}\right]^{\frac{1}{\sigma_s}}.$$

Corollary 1 implies that

$$\begin{aligned} & mpg^E_{s,j,t+1} = \tilde{R}^p_s \\ \iff & \theta_{s,j}\gamma\gamma_s\frac{Y_{t+1}}{G^E_{t+1}}\frac{G^E_{t+1}}{G^E_{s,t+1}}\left(\frac{G^E_{s,t+1}}{g^E_{s,j,t+1}}\right)^{\frac{1}{\sigma_s}} = \tilde{R}^p_s \\ \iff & g^{E*}_{s,j,t+1} = \left(\frac{\theta_{s,j}\gamma\gamma_s}{\tilde{R}^p_s}Y_t\right)^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s} \end{aligned}$$

And the effective profit is

$$\begin{aligned} \tilde{\pi}^p_s &= \int_0^{g^{E*}_{s,j,t+1}}(mpg^E_{s,j,t+1}-\tilde{R}^p_s)dg^E_{s,j,t+1} - f^p_s \\ &= \int_0^{g^{E*}_{s,j,t+1}}\left(\theta_{s,j}\gamma\gamma_s\frac{Y_{t+1}}{G^E_{s,t+1}}\left(\frac{G^E_{s,t+1}}{g^E_{s,j,t+1}}\right)^{\frac{1}{\sigma_s}} - \tilde{R}^p_s\right)dg^E_{s,j,t+1} - f^p_s \\ &= \theta_{s,j}\gamma\gamma_s Y_{t+1}(G^E_{s,t+1})^{\frac{1-\sigma_s}{\sigma_s}}\int_0^{g^{E*}_{s,j,t+1}}(g^E_{s,j,t+1})^{-\frac{1}{\sigma_s}}dg^E_{s,j,t+1} - \tilde{R}^p_s g^{E*}_{s,j,t+1} - f^p_s \\ &= \theta_{s,j}\gamma\gamma_s Y_{t+1}(G^E_{s,t+1})^{\frac{1-\sigma_s}{\sigma_s}}\frac{\sigma_s}{\sigma_s-1}(g^{E*}_{s,j,t+1})^{\frac{\sigma_s-1}{\sigma_s}} - \tilde{R}^p_s g^{E*}_{s,j,t+1} - f^p_s \\ &= \frac{\sigma_s}{\sigma_s-1}\left(\theta_{s,j}\gamma\gamma_s Y_{t+1}\right)^{\sigma_s}(\tilde{R}^p_s G^E_{s,t+1})^{1-\sigma_s} - \left(\theta_{s,j}\gamma\gamma_s Y_{t+1}\right)^{\sigma_s}(\tilde{R}^p_s G^E_{s,t+1})^{1-\sigma_s} - f^p_s \\ &= \frac{1}{\sigma_s-1}\left(\theta_{s,j}\gamma\gamma_s Y_{t+1}\right)^{\sigma_s}(\tilde{R}^p_s G^E_{s,t+1})^{1-\sigma_s} - f^p_s \end{aligned}$$

Zero-profit cutoff can be obtained by equating $\tilde{\pi}^p_s$ to zero.

$$\begin{aligned} & \tilde{\pi}^p_s(\bar{\theta}^p_{s,t+1}) = 0 \\ \iff & \bar{\theta}^p_{s,t+1} = \frac{\left((\sigma_s-1)f^p_s\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^p_s)^{\frac{\sigma_s-1}{\sigma_s}} \end{aligned}$$

Now, I compare $\tilde{\pi}^p_s(\theta)$ and $\tilde{\pi}^{p'}_s(\theta)$. Let's define the difference function $diff(\theta) \equiv \tilde{\pi}^p_s(\theta) -$

$\tilde{\pi}_s^{p'}(\theta)$.

$$
\begin{aligned}
diff(\theta) &= \frac{1}{\sigma_s - 1}\left(\theta\gamma\gamma_s Y_{t+1}\right)^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}((\tilde{R}^p_s)^{1-\sigma_s} - (\tilde{R}^{p'}_s)^{1-\sigma_s}) - (f^p_s - f^{p'}_s). \\
&= \frac{1}{\sigma_s - 1}\left(\theta\gamma\gamma_s Y_{t+1}\right)^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}(\tilde{R}^p_s\tilde{R}^{p'}_s)^{1-\sigma_s}\left((\tilde{R}^{p'}_s)^{\sigma_s-1} - (\tilde{R}^p_s)^{\sigma_s-1}\right) - (f^p_s - f^{p'}_s)
\end{aligned}
$$

Suppose that $\tilde{R}^{p'}_s > \tilde{R}^p_s$. Then, $diff(\theta)$ is strictly increasing in θ.Let's first find the productivity $\bar{\theta}^I_{s,t+1}$ that makes the difference zero so the government is indifferent between p and p'.

$$
\begin{aligned}
& diff(\bar{\theta}^I_{s,t}) = 0 \\
& \iff \bar{\theta}^I_{s,t} = \frac{\left((\sigma_s - 1)(f^p_s - f^{p'}_s)\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^p_s\tilde{R}^{p'}_s)^{\frac{\sigma_s-1}{\sigma_s}}\left[\frac{1}{(\tilde{R}^{p'}_s)^{\sigma_s-1} - (\tilde{R}^p_s)^{\sigma_s-1}}\right]^{\frac{1}{\sigma_s}}
\end{aligned}
$$

The cutoff is well-defined only if $f^p_s > f^{p'}_s$. Otherwise, the difference is always positive hence it is optimal to choose p over p' for all θ. If $f^p_s > f^{p'}_s$, for all $\theta > \bar{\theta}^I_{s,t+1}$, $\tilde{\pi}^p_s(\theta) > \tilde{\pi}^{p'}_s(\theta)$ while for all $\theta \leq \bar{\theta}^I_{s,t+1}$, $\tilde{\pi}^p_s(\theta) \leq \tilde{\pi}^{p'}_s(\theta)$. In sector s, for there to be any active project that is financed by p', the cutoffs should be such that $\bar{\theta}^{p'}_s < \bar{\theta}^I_s$.

$$
\begin{aligned}
& \bar{\theta}^{p'}_s < \bar{\theta}^I_s \\
\iff & \frac{\left((\sigma_s - 1)f^{p'}_s\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^{p'}_s)^{\frac{\sigma_s-1}{\sigma_s}} < \frac{\left((\sigma_s - 1)(f^p_s - f^{p'}_s)\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^p_s\tilde{R}^{p'}_s)^{\frac{\sigma_s-1}{\sigma_s}}\left[\frac{1}{(\tilde{R}^{p'}_s)^{\sigma_s-1} - (\tilde{R}^p_s)^{\sigma_s-1}}\right]^{\frac{1}{\sigma_s}} \\
\iff & f^{p'}_s < (f^p_s - f^{p'}_s)(\tilde{R}^p_s)^{\sigma_s-1}\frac{1}{(\tilde{R}^{p'}_s)^{\sigma_s-1} - (\tilde{R}^p_s)^{\sigma_s-1}} \\
\iff & f^{p'}_s((\tilde{R}^{p'}_s)^{\sigma_s-1} - (\tilde{R}^p_s)^{\sigma_s-1}) < (f^p_s - f^{p'}_s)(\tilde{R}^p_s)^{\sigma_s-1} \\
\iff & f^{p'}_s(\tilde{R}^{p'}_s)^{\sigma_s-1} < f^p_s(\tilde{R}^p_s)^{\sigma_s-1} \\
\iff & \left(\frac{\tilde{R}^{p'}_s}{\tilde{R}^p_s}\right)^{\sigma_s-1} f^{p'}_s < f^p_s.
\end{aligned}
$$

Hence, if $f^p_s \leq \left(\frac{\tilde{R}^{p'}_s}{\tilde{R}^p_s}\right)^{\sigma_s-1} f^{p'}_s$, all projects that make a positive effective profit when financed by p' can make a higher profit when financed by p. Therefore, all operating projects in sector s is financed by p. If $f^p_s > \left(\frac{\tilde{R}^{p'}_s}{\tilde{R}^p_s}\right)^{\sigma_s-1} f^{p'}_s$, projects with $\theta \geq \bar{\theta}^I_{s,t+1}$ are financed by p and projects with $\theta \in [\bar{\theta}^{p'}_{s,t+1}, \bar{\theta}^I_{s,t+1})$ are financed by p'. □

3.6 Full Statement and Proof of Proposition 2

Let $S^{pp'}$ denote the set of sectors where projects with $\theta \geq \bar{\theta}^I$ are financed by p, and projects with $\theta < \bar{\theta}^I$ are financed by p'. And let S^p denote the set of sectors where all projects with $\theta \geq \bar{\theta}^p$ are financed by p. A superscript with a tilde ($\tilde{}$) indicates maximal diversion, while a superscript without a tilde indicates zero diversion. Each sector falls into one of the following seven categories based on corruption levels and fixed costs:

	$\chi < R_s^D$	$R_s^D < \chi < R_s^C$	$R_s^C < \chi < \frac{\psi_s^D R_s^C - \psi_s^C R_s^D}{\psi_s^D - \psi_s^C}$	$\frac{\psi_s^D R_s^C - \psi_s^C R_s^D}{\psi_s^D - \psi_s^C} < \chi$
$f_s^D \leq \left(\frac{\tilde{R}_s^C}{\tilde{R}_s^D}\right)^{\sigma_s - 1} f_s^C$	$s \in S^D$	$s \in S^{\tilde{D}}$	$s \in S^{\tilde{D}}$	$s \in S^{\tilde{C}\tilde{D}}$
$f_s^D > \left(\frac{\tilde{R}_s^C}{\tilde{R}_s^D}\right)^{\sigma_s - 1} f_s^C$	$s \in S^{DC}$	$s \in S^{\tilde{D}C}$	$s \in S^{\tilde{D}\tilde{C}}$	$s \in S^{\tilde{C}}$

First, suppose $\chi < R_s^D < R_s^C$. Lemma 2 implies that it is optimal to choose zero diversion for both DAC and China. Then, $\tilde{R}_s^D = R_s^D - (1 - \delta_s^E) < R_s^C - (1 - \delta_s^E) = \tilde{R}_s^C$. Lemma 4 implies that if $f_s^D \leq (\frac{\tilde{R}_s^C}{\tilde{R}_s^D})^{\sigma_s - 1} f_s^C$, all projects in sector s are financed by DAC and hence $s \in S^D$ while if $f_s^D > (\frac{\tilde{R}_s^C}{\tilde{R}_s^D})^{\sigma_s - 1} f_s^C$, projects with $\theta \geq \bar{\theta}_{s,t+1}^I$ are financed by DAC and projects with $\theta \in [\bar{\theta}_{s,t+1}^C, \bar{\theta}_{s,t+1}^I)$ are financed by China and hence $s \in S^{DC}$.

Second, suppose $R_s^D < \chi < R_s^C$. Lemma 2 implies that the government chooses maximal diversion for DAC and zero diversion for China. Then, $\tilde{R}_s^D = \frac{R_s^D - (1-\psi_s^D)\chi}{\psi_s^D} - (1 - \delta_s^E) < R_s^C - (1 - \delta_s^E) = \tilde{R}_s^C$. Lemma 4 implies that if f_s^D is not greater than the threshold, all projects are financed by DAC so $s \in S^{\tilde{D}}$ while if f_s^D is greater than the threshold, projects with $\theta \geq \bar{\theta}_{s,t+1}^I$ are financed by DAC and projects with $\theta \in [\bar{\theta}_{s,t+1}^C, \bar{\theta}_{s,t+1}^I)$ are financed by China and hence $s \in S^{\tilde{D}C}$.

Third, suppose $R_s^D < R_s^C < \chi < \frac{\psi_s^D R_s^C - \psi_s^C R_s^D}{\psi_s^D - \psi_s^C}$. By Lemma 2, the government chooses maximal diversion for both DAC and China. Since $\chi < \frac{\psi_s^D R_s^C - \psi_s^C R_s^D}{\psi_s^D - \psi_s^C}$, $\tilde{R}_s^D = \frac{R_s^D - (1-\psi_s^D)\chi}{\psi_s^D} - (1 - \delta_s^E) < \frac{R_s^C - (1-\psi_s^C)\chi}{\psi_s^C} - (1 - \delta_s^E) = \tilde{R}_s^C$. The rest follows a similar logic to the one used for the above two cases.

Lastly, suppose $\frac{\psi_s^D R_s^C - \psi_s^C R_s^D}{\psi_s^D - \psi_s^C} < \chi$. By Lemma2, the government chooses maximal diversion for both DAC and China. However, $\tilde{R}_s^D > \tilde{R}_s^C$. Hence, Lemma 4 implies that if $f_s^C \leq (\frac{\tilde{R}_s^D}{\tilde{R}_s^C})^{\sigma_s - 1} f_s^D$, all projects are financed by China so $s \in S^{\tilde{C}}$. If $f_s^C > (\frac{\tilde{R}_s^D}{\tilde{R}_s^C})^{\sigma_s - 1} f_s^D$, projects with $\theta \geq \bar{\theta}_{s,t+1}^I$ are financed by China and projects with $\theta \in [\bar{\theta}_{s,t+1}^D, \bar{\theta}_{s,t+1}^I)$ are financed by DAC so $s \in S^{\tilde{C}\tilde{D}}$. $\square$

3.7 Full Statement and Proof of Proposition 3

The effective public capital in sector s for period t is given by:

$$G^E_{s,t} = \mathcal{G}^E_s \cdot Y_t^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}},$$

where

$$\mathcal{G}^E_s = \begin{cases} \mathcal{G}^{E,D}_s \cdot \mathcal{G}_s & \text{if } s \in (S^D \cup S^{\tilde{D}}) \\ \mathcal{G}^{E,C}_s \cdot \mathcal{G}_s & \text{if } s \in S^{\tilde{C}} \\ \mathcal{G}^{E,DC}_s \cdot \mathcal{G}_s & \text{if } s \in (S^{DC} \cup S^{\tilde{D}C} \cup S^{\tilde{D}\tilde{C}}) \\ \mathcal{G}^{E,CD}_s \cdot \mathcal{G}_s & \text{if } s \in S^{\tilde{C}\tilde{D}}. \end{cases}$$

Here, $\mathcal{G}_s$ is a factor not related to the financing choices, defined as:

$$\mathcal{G}_s \equiv \big(\sigma_s - 1\big)^{\frac{\sigma_s-\xi_s}{\xi_s(\sigma_s-1)}} \big(\gamma\gamma_s\big)^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} \left(\frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s-\sigma_s}\right)^{\frac{\sigma_s}{\xi_s(\sigma_s-1)}}$$

and the other financing-specific factors are:

$$\begin{aligned}
\mathcal{G}^{E,D}_s &\equiv (\tilde{R}^D_s)^{-1}(f^D_s)^{\frac{\sigma_s-\xi_s}{\xi_s(\sigma_s-1)}}, \\
\mathcal{G}^{E,C}_s &\equiv (\tilde{R}^C_s)^{-1}(f^C_s)^{\frac{\sigma_s-\xi_s}{\xi_s(\sigma_s-1)}}, \\
\mathcal{G}^{E,DC}_s &\equiv \left[f^C_s\Big(\frac{(\tilde{R}^C_s)^{1-\sigma_s}}{f^C_s}\Big)^{\frac{\xi_s}{\sigma_s}} + (f^D_s - f^C_s)\Big(\frac{(\tilde{R}^D_s)^{1-\sigma_s} - (\tilde{R}^C_s)^{1-\sigma_s}}{f^D_s - f^C_s}\Big)^{\frac{\xi_s}{\sigma_s}}\right]^{\frac{\sigma_s}{\xi_s(\sigma_s-1)}} \\
\mathcal{G}^{E,CD}_s &\equiv \left[f^D_s\Big(\frac{(\tilde{R}^D_s)^{1-\sigma_s}}{f^D_s}\Big)^{\frac{\xi_s}{\sigma_s}} + (f^C_s - f^D_s)\Big(\frac{(\tilde{R}^C_s)^{1-\sigma_s} - (\tilde{R}^D_s)^{1-\sigma_s}}{f^C_s - f^D_s}\Big)^{\frac{\xi_s}{\sigma_s}}\right]^{\frac{\sigma_s}{\xi_s(\sigma_s-1)}}.
\end{aligned}$$

Suppose that sector s is financed by a single provider, say p. Corollary 1 implies that the optimal project size for each j in sector s is $g^{E*}_{s,j,t+1} = (\theta_{s,j}\gamma\gamma_s Y_{t+1}/\tilde{R}^p_s)^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}$.

Plugging this into the definition of $G^E_{s,t+1}$, I get

$$
\begin{aligned}
G^E_{s,t+1} &= \left[\int_{j\in J_s} \theta_{s,j} {g^E_{s,j,t+1}}^{\frac{\sigma_s-1}{\sigma_s}} dj\right]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= \left[\int_{\theta_s} \theta_s {g^E_{s,j,t+1}}^{\frac{\sigma_s-1}{\sigma_s}} dH_s(\theta_s)\right]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= \left[\int_{\theta_s} \theta_s \left((\frac{\theta_s \gamma\gamma_s Y_{t+1}}{\tilde{R}^p_s})^{\sigma_s} (G^E_{s,t+1})^{1-\sigma_s} \right)^{\frac{\sigma_s-1}{\sigma_s}} dH_s(\theta_s)\right]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\frac{\gamma\gamma_s Y_{t+1}}{\tilde{R}^p_s})^{\sigma_s} (G^E_{s,t+1})^{1-\sigma_s} (\xi_s {\theta^s_{min}}^{\xi_s})^{\frac{\sigma_s}{\sigma_s-1}} \left[\int_{\bar{\theta}^p_{s,t+1}}^{\infty} \theta_s^{\sigma_s-\xi_s-1} d\theta_s\right]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\frac{\gamma\gamma_s Y_{t+1}}{\tilde{R}^p_s})^{\sigma_s} (G^E_{s,t+1})^{1-\sigma_s} (\xi_s {\theta^s_{min}}^{\xi_s})^{\frac{\sigma_s}{\sigma_s-1}} \left[\frac{1}{\sigma_s-\xi_s} \theta_s^{\sigma_s-\xi_s} \Big|_{\bar{\theta}^p_{s,t+1}}^{\infty}\right]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\frac{\gamma\gamma_s Y_{t+1}}{\tilde{R}^p_s})^{\sigma_s} (G^E_{s,t+1})^{1-\sigma_s} (\frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\sigma_s-1}} \left[\left(\frac{((\sigma_s-1)f^p_s)^{\frac{\sigma_s-\xi_s}{\sigma_s}}}{(\gamma\gamma_s Y_{t+1})^{\sigma_s-\xi_s}} (G^E_{s,t+1}\tilde{R}^p_s)^{\frac{(\sigma_s-1)(\sigma_s-\xi_s)}{\sigma_s}} \right)\right]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\frac{\sigma_s(\xi_s-1)}{\sigma_s-1}} (\tilde{R}^p_s)^{-\xi_s} (G^E_{s,t+1})^{1-\xi_s} ((\sigma_s-1)f^p_s)^{\frac{\sigma_s-\xi_s}{\sigma_s-1}} (\frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\sigma_s-1}}
\end{aligned}
$$

Rearranging,

$$
G^E_{s,t+1} = \frac{((\sigma_s-1)f^p_s)^{\frac{\sigma_s-\xi_s}{\xi_s(\sigma_s-1)}}}{\tilde{R}^p_s} (\frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\xi_s(\sigma_s-1)}}) (\gamma\gamma_s Y_{t+1})^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}}
$$

Now, suppose that sector s is financed by both p and p' and $\tilde{R}^p_s < \tilde{R}^{p'}_s$. Lemma 5 implies that projects with $\theta \geq \bar{\theta}^I_{s,t+1}$ are financed by p and projects with $\theta \in [\bar{\theta}^{p'}_{s,t+1}, \bar{\theta}^I_{s,t+1})$ are financed by p'. Then,

$$
\begin{aligned}
G^E_{s,t+1} &= \Big[\int_{j\in J_s} \theta_{s,j} g^E_{s,j,t+1}{}^{\frac{\sigma_s-1}{\sigma_s}} dj\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= \Big[\int_{\theta_s} \theta_s g^E_{s,j,t+1}{}^{\frac{\sigma_s-1}{\sigma_s}} dH_s(\theta_s)\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= \Big[\int_{\bar{\theta}^{p'}_{s,t+1}}^{\bar{\theta}^I_{s,t+1}} \theta_s g^E_{s,j,t+1}{}^{\frac{\sigma_s-1}{\sigma_s}} dH_s(\theta_s) + \int_{\bar{\theta}^I_{s,t+1}}^{\infty} \theta_s g^E_{s,j,t+1}{}^{\frac{\sigma_s-1}{\sigma_s}} dH_s(\theta_s)\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}(\xi_s \theta^s_{min}{}^{\xi_s})^{\frac{\sigma_s}{\sigma_s-1}}\Big[(\tilde{R}^{p'}_s)^{1-\sigma_s}\int_{\bar{\theta}^{p'}_{s,t+1}}^{\bar{\theta}^I_{s,t+1}} \theta_s^{\sigma_s-\xi_s-1} d\theta_s + (\tilde{R}^p_s)^{1-\sigma_s}\int_{\bar{\theta}^I_{s,t+1}}^{\infty} \theta_s^{\sigma_s-\xi_s-1} d\theta_s\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}(\xi_s \theta^s_{min}{}^{\xi_s})^{\frac{\sigma_s}{\sigma_s-1}}\Big[\frac{(\tilde{R}^{p'}_s)^{1-\sigma_s}}{\sigma_s-\xi_s}\theta_s^{\sigma_s-\xi_s}\Big|_{\bar{\theta}^{p'}_{s,t+1}}^{\bar{\theta}^I_{s,t+1}} + \frac{(\tilde{R}^p_s)^{1-\sigma_s}}{\sigma_s-\xi_s}\theta_s^{\sigma_s-\xi_s}\Big|_{\bar{\theta}^I_{s,t+1}}^{\infty}\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}(\xi_s \theta^s_{min}{}^{\xi_s})^{\frac{\sigma_s}{\sigma_s-1}}\Big[\frac{(\tilde{R}^{p'}_s)^{1-\sigma_s}}{\sigma_s-\xi_s}((\bar{\theta}^I_{s,t+1})^{\sigma_s-\xi_s} - (\bar{\theta}^{p'}_{s,t+1})^{\sigma_s-\xi_s}) - \frac{(\tilde{R}^p_s)^{1-\sigma_s}}{\sigma_s-\xi_s}(\bar{\theta}^I_{s,t+1})^{\sigma_s-\xi_s}\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\sigma_s}(G^E_{s,t+1})^{1-\sigma_s}(\frac{\xi_s \theta^s_{min}{}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\sigma_s-1}}\Big[((\tilde{R}^p_s)^{1-\sigma_s} - (\tilde{R}^{p'}_s)^{1-\sigma_s})(\bar{\theta}^I_{s,t+1})^{\sigma_s-\xi_s} + (\tilde{R}^{p'}_s)^{1-\sigma_s}(\bar{\theta}^{p'}_{s,t+1})^{\sigma_s-\xi_s}\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\frac{\sigma_s(\xi_s-1)}{\sigma_s-1}}(G^E_{s,t+1})^{1-\xi_s}(\sigma_s-1)^{\frac{\sigma_s-\xi_s}{\sigma_s-1}}(\frac{\xi_s \theta^s_{min}{}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\sigma_s-1}} \\
&\quad\times\Big[(f^p_s - f^{p'}_s)^{\frac{\sigma_s-\xi_s}{\sigma_s}}((\tilde{R}^p_s)^{1-\sigma_s} - (\tilde{R}^{p'}_s)^{1-\sigma_s})^{\frac{\xi_s}{\sigma_s}} + (f^{p'}_s)^{\frac{\sigma_s-\xi_s}{\sigma_s}}(\tilde{R}^{p'}_s)^{\frac{\xi_s(1-\sigma_s)}{\sigma_s}}\Big]^{\frac{\sigma_s}{\sigma_s-1}} \\
&= (\gamma\gamma_s Y_{t+1})^{\frac{\sigma_s(\xi_s-1)}{\sigma_s-1}}(G^E_{s,t+1})^{1-\xi_s}(\sigma_s-1)^{\frac{\sigma_s-\xi_s}{\sigma_s-1}}(\frac{\xi_s \theta^s_{min}{}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\sigma_s-1}} \\
&\quad\times\Big[f^{p'}_s(\frac{(\tilde{R}^{p'}_s)^{1-\sigma_s}}{f^{p'}_s})^{\frac{\xi_s}{\sigma_s}} + (f^p_s - f^{p'}_s)(\frac{(\tilde{R}^p_s)^{1-\sigma_s} - (\tilde{R}^{p'}_s)^{1-\sigma_s}}{f^p_s - f^{p'}_s})^{\frac{\xi_s}{\sigma_s}}\Big]^{\frac{\sigma_s}{\sigma_s-1}}
\end{aligned}
$$

Rearranging,

$$
\begin{aligned}
G^E_{s,t+1} = &\Big[f^{p'}_s(\frac{(\tilde{R}^{p'}_s)^{1-\sigma_s}}{f^{p'}_s})^{\frac{\xi_s}{\sigma_s}} + (f^p_s - f^{p'}_s)(\frac{(\tilde{R}^p_s)^{1-\sigma_s} - (\tilde{R}^{p'}_s)^{1-\sigma_s}}{f^p_s - f^{p'}_s})^{\frac{\xi_s}{\sigma_s}}\Big]^{\frac{\sigma_s}{\xi_s(\sigma_s-1)}} \\
&\times(\sigma_s-1)^{\frac{\sigma_s-\xi_s}{\xi_s(\sigma_s-1)}}(\frac{\xi_s \theta^s_{min}{}^{\xi_s}}{\xi_s-\sigma_s})^{\frac{\sigma_s}{\xi_s(\sigma_s-1)}}(\gamma\gamma_s Y_{t+1})^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}}
\end{aligned}
$$

Proposition 2 implies that all sectors fall into one of the two cases. Sectors in $S^D \cup S^{\tilde{D}} \cup S^{\tilde{C}}$ correspond to the first case and sectors in $S^{DC} \cup S^{\tilde{D}C} \cup S^{\tilde{D}\tilde{C}} \cup S^{\tilde{C}\tilde{D}}$ correspond to the second case. Replacing p and p' with D and C accordingly concludes the proof. □

3.8 Proof of Proposition 4

$$
\begin{aligned}
G_t^E &= \prod_{s \in S} (G_{s,t}^E)^{\gamma_s} \\
&= \prod_{s \in S} (\mathcal{G}_s^E Y_t^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}})^{\gamma_s} \\
&= \left(\prod_{s \in S} (\mathcal{G}_s^E)^{\gamma_s} \right) Y_t^{\sum_s \frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)} \gamma_s} \\
&= \mathcal{G}^E Y_t^{\sum_s \frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)} \gamma_s}
\end{aligned}
$$

□

3.9 Full Statement and Proof of Proposition 5

The expected observed size of a project financed by p in sector s is given by:

$$\mathbb{E}[g^O_{s,j,t}|p,s] = \frac{\xi_s(\sigma-1)}{\Psi^p_s \tilde{R}^p_s(\xi_s-\sigma)}\mathcal{F}^p_s.$$

$\mathcal{F}^p_s$ is defined as follows:

- DAC grants

$$\mathcal{F}^p_s = \begin{cases} \tilde{f}^G_s & \text{if } s \in \{S^G, S^{\tilde{G}}\} \\ \frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}}{(\tilde{f}^G_s)^{-\frac{\xi_s}{\sigma}} - (f^D_s)^{-\frac{\xi_s}{\sigma}}} & \text{if } s \in \{S^D, S^{\tilde{D}}, S^{\tilde{C}\tilde{D}}\} \\ \frac{1}{(\tilde{R}^D_s)^{\sigma-1}} \frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}}{(\tilde{f}^G_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}} & \text{if } s \in \{S^{\tilde{C}}, S^{\tilde{D}\tilde{C}}, S^{\tilde{D}C}, S^{DC}\} \end{cases}$$

- DAC loans

$$\mathcal{F}^p_s = \begin{cases} f^D_s & \text{if } s \in \{S^D, S^{\tilde{D}}\} \\ \frac{(f^D_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\right)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^D_s)^{\frac{-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\right)^{\frac{-\xi_s}{\sigma}}} & \text{if } s \in \{S^{\tilde{C}\tilde{D}}\} \\ (f^D_s - f^C_s)\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}} & \text{if } s \in \{S^{DC}, S^{\tilde{D}C}, S^{\tilde{D}\tilde{C}}\} \end{cases}$$

- Chinese loans

$$\mathcal{F}^p_s = \begin{cases} f^C_s & \text{if } s \in \{S^C, S^{\tilde{C}}\} \\ \frac{(f^C_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^C_s)^{\frac{-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{-\xi_s}{\sigma}}} & \text{if } s \in \{S^{DC}, S^{\tilde{D}C}, S^{\tilde{D}\tilde{C}}\} \\ (f^C_s - f^D_s)\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}} & \text{if } s \in \{S^{\tilde{C}\tilde{D}}\} \end{cases}$$

3.9.1 DAC Grants

(1) If $s \in \{S^G, S^{\tilde{G}}\}$
It is convenient to define $\tilde{f}_s^G \equiv \frac{f_s^G}{1+(\sigma-1)\frac{R_s^D}{\Psi_s^D \tilde{R}_s^D}}$. The expected size of grant-financed projects is

$$
\begin{aligned}
\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^G_{s,t} \leq \theta_j\Big] &= \mathbb{E}\Big[\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^G_{s,t} \leq \theta_j, Y_t, G^E_{s,t}\Big]\Big] \quad \text{(by Law of Iterated Expectation)} \\
&= \mathbb{E}\Big[\mathbb{E}\Big[\frac{1}{\Psi_s^D}(\frac{\theta_j \gamma\gamma_s Y_t}{\tilde{R}_s^D})^\sigma (G^E_{s,t})^{1-\sigma}\Big|\bar{\theta}^G_{s,t} \leq \theta_j, Y_t, G^E_{s,t}\Big]\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi_s^D}(\frac{\gamma\gamma_s Y_t}{\tilde{R}_s^D})^\sigma (G^E_{s,t})^{1-\sigma}\mathbb{E}\Big[\theta^\sigma\Big|\bar{\theta}^G_{s,t} \leq \theta_j, Y_t, G^E_{s,t}\Big]\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi_s^D}(\frac{\gamma\gamma_s Y_t}{\tilde{R}_s^D})^\sigma (G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^G_{s,t}}^\infty \theta^\sigma \frac{h(\theta)}{H(\infty)-H(\bar{\theta}^G_{s,t})}d\theta\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi_s^D}(\frac{\gamma\gamma_s Y_t}{\tilde{R}_s^D})^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s \underline{\theta}^{\xi_s}}{\xi_s-\sigma}\frac{(\bar{\theta}^G_{s,t})^{\xi_s}}{\underline{\theta}^{\xi_s}}\Big((\bar{\theta}^G_{s,t})^{\sigma-\xi_s}\Big)\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi_s^D}(\frac{\gamma\gamma_s Y_t}{\tilde{R}_s^D})^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s-\sigma}\Big(\frac{\big((\sigma-1)\tilde{f}_s^G\big)^{1/\sigma}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}_s^D)^{\frac{\sigma-1}{\sigma}}\Big)^\sigma\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi_s^D}\frac{1}{\tilde{R}_s^D}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)\tilde{f}_s^G\Big] \\
&= \mathbb{E}\Big[\frac{\xi_s(\sigma-1)}{\Psi_s^D\tilde{R}_s^D(\xi_s-\sigma)}\tilde{f}_s^G\Big] \\
&= \frac{\xi_s(\sigma-1)}{\Psi_s^D\tilde{R}_s^D(\xi_s-\sigma)}\tilde{f}_s^G
\end{aligned}
$$

(2) If $s \in \{S^D, S^{\tilde{D}}, S^{\tilde{C}\tilde{D}}\}$

$$
\begin{aligned}
&\mathbb{E}\left[g^O_{s,j,t}\middle|\bar{\theta}^G_{s,t} \leq \theta_j \leq \bar{\theta}^D_{s,t}\right] \\
&\qquad = \mathbb{E}\left[\mathbb{E}\left[g^O_{s,j,t}\middle|\bar{\theta}^G_{s,t} \leq \theta_j \leq \bar{\theta}^D_{s,t}, Y_t, G^E_{s,t}\right]\right] \quad \text{(by Law of Iterated Expectation)} \\
&\qquad = \mathbb{E}\left[\mathbb{E}\left[\frac{1}{\Psi^D_s}\left(\frac{\theta_j\gamma\gamma_s Y_t}{\tilde{R}^D_s}\right)^\sigma (G^E_{s,t})^{1-\sigma}\middle|\bar{\theta}^G_{s,t} \leq \theta_j \leq \bar{\theta}^D_{s,t}, Y_t, G^E_{s,t}\right]\right] \\
&\qquad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\left(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\right)^\sigma (G^E_{s,t})^{1-\sigma}\mathbb{E}\left[\theta^\sigma\middle|\bar{\theta}^G_{s,t} \leq \theta_j \leq \bar{\theta}^D_{s,t}, Y_t, G^E_{s,t}\right]\right] \\
&\qquad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\left(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\right)^\sigma (G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^G_{s,t}}^{\bar{\theta}^D_{s,t}} \theta^\sigma \frac{h(\theta)}{H(\bar{\theta}^D_{s,t}) - H(\bar{\theta}^G_{s,t})} d\theta\right] \\
&\qquad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\left(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\right)^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s\underline{\theta}^{\xi_s}}{\xi_s-\sigma}\frac{1}{\underline{\theta}^{\xi_s}}\frac{1}{(\bar{\theta}^G_{s,t})^{-\xi_s} - (\bar{\theta}^D_{s,t})^{-\xi_s}}\left((\bar{\theta}^G_{s,t})^{\sigma-\xi_s} - (\bar{\theta}^D_{s,t})^{\sigma-\xi_s}\right)\right] \\
&\qquad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\left(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\right)^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s-\sigma}\right. \\
&\qquad\qquad \left.\times \frac{(\sigma-1)^{\frac{\sigma-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s-\sigma}(G^E_{s,t})^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}\left((\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}\right)}{(\sigma-1)^{\frac{-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s}(G^E_{s,t})^{-\frac{\xi_s(\sigma-1)}{\sigma}}\left((\tilde{f}^G_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^D_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}\right)}\right] \\
&\qquad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\frac{1}{\tilde{R}^D_s}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)\frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}}{(\tilde{f}^G_s)^{-\frac{\xi_s}{\sigma}} - (f^D_s)^{-\frac{\xi_s}{\sigma}}}\right] \\
&\qquad = \mathbb{E}\left[\frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}}{(\tilde{f}^G_s)^{-\frac{\xi_s}{\sigma}} - (f^D_s)^{-\frac{\xi_s}{\sigma}}}\right] \\
&\qquad = \frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}}{(\tilde{f}^G_s)^{-\frac{\xi_s}{\sigma}} - (f^D_s)^{-\frac{\xi_s}{\sigma}}}
\end{aligned}
$$

(3) If $s \in \{S^{\tilde{C}}, S^{\tilde{D}\tilde{C}}, S^{\tilde{D}C}, S^{DC}\}$

$$
\begin{aligned}
&\mathbb{E}\left[g^O_{s,j,t}\middle|\bar{\theta}^G_{s,t} \le \theta_j \le \bar{\theta}^C_{s,t}\right] \\
&\quad = \mathbb{E}\left[\mathbb{E}\left[g^O_{s,j,t}\middle|\bar{\theta}^G_{s,t} \le \theta_j \le \bar{\theta}^D_{s,t}, Y_t, G^E_{s,t}\right]\right] \quad \text{(by Law of Iterated Expectation)} \\
&\quad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\Big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\Big)^\sigma (G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^G_{s,t}}^{\bar{\theta}^C_{s,t}} \theta^\sigma \frac{h(\theta)}{H(\bar{\theta}^C_{s,t}) - H(\bar{\theta}^G_{s,t})} d\theta\right] \\
&\quad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\Big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\Big)^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s \underline{\theta}^{\xi_s}}{\xi_s - \sigma}\frac{1}{\underline{\theta}^{\xi_s}}\frac{1}{(\bar{\theta}^G_{s,t})^{-\xi_s} - (\bar{\theta}^C_{s,t})^{-\xi_s}}\Big((\bar{\theta}^G_{s,t})^{\sigma-\xi_s} - (\bar{\theta}^C_{s,t})^{\sigma-\xi_s}\Big)\right] \\
&\quad = \mathbb{E}\Bigg[\frac{1}{\Psi^D_s}\Big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\Big)^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s - \sigma} \\
&\qquad \times \frac{(\sigma-1)^{\frac{\sigma-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s-\sigma}(G^E_{s,t})^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}\Big((\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}\Big)}{(\sigma-1)^{\frac{-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s}(G^E_{s,t})^{-\frac{\xi_s(\sigma-1)}{\sigma}}\Big((\tilde{f}^G_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}\Big)}\Bigg] \\
&\quad = \mathbb{E}\left[\frac{1}{\Psi^D_s}\frac{1}{\tilde{R}^D_s}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)\frac{1}{(\tilde{R}^D_s)^{\sigma-1}}\frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}}{(\tilde{f}^G_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}}\right] \\
&\quad = \mathbb{E}\left[\frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{1}{(\tilde{R}^D_s)^{\sigma-1}}\frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}}{(\tilde{f}^G_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}}\right] \\
&\quad = \frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{1}{(\tilde{R}^D_s)^{\sigma-1}}\frac{(\tilde{f}^G_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}}{(\tilde{f}^G_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}}
\end{aligned}
$$

3.9.2 DAC Loans

(1) If $s \in \{S^D, S^{\tilde{D}}\}$

$$
\begin{aligned}
\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^D_{s,t} \leq \theta_j\Big] &= \mathbb{E}\Big[\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^D_{s,t} \leq \theta_j, Y_t, G^E_{s,t}\Big]\Big] \quad \text{(by Law of Iterated Expectation)} \\
&= \mathbb{E}\Big[\mathbb{E}\Big[\frac{1}{\Psi^D_s}(\frac{\theta_j \gamma\gamma_s Y_t}{\tilde{R}^D_s})^\sigma (G^E_{s,t})^{1-\sigma}\Big|\bar{\theta}^D_{s,t} \leq \theta_j, Y_t, G^E_{s,t}\Big]\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^D_s}(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s})^\sigma (G^E_{s,t})^{1-\sigma}\mathbb{E}\Big[\theta^\sigma\Big|\bar{\theta}^D_{s,t} \leq \theta_j, Y_t, G^E_{s,t}\Big]\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^D_s}(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s})^\sigma (G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^D_{s,t}}^{\infty} \theta^\sigma \frac{h(\theta)}{H(\infty) - H(\bar{\theta}^D_{s,t})} d\theta\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^D_s}(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s})^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s \underline{\theta}^{\xi_s}}{\xi_s - \sigma}\frac{(\bar{\theta}^D_{s,t})^{\xi_s}}{\underline{\theta}^{\xi_s}}\Big((\bar{\theta}^D_{s,t})^{\sigma-\xi_s}\Big)\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^D_s}(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s})^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s - \sigma}\Big(\frac{\big((\sigma-1)f^D_s\big)^{1/\sigma}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^D_s)^{\frac{\sigma-1}{\sigma}}\Big)^\sigma\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^D_s}\frac{1}{\tilde{R}^D_s}\frac{\xi_s}{\xi_s - \sigma}(\sigma-1)f^D_s\Big] \\
&= \mathbb{E}\Big[\frac{\xi_s(\sigma-1)}{\Psi^D_s \tilde{R}^D_s(\xi_s - \sigma)}f^D_s\Big] \\
&= \frac{\xi_s(\sigma-1)}{\Psi^D_s \tilde{R}^D_s(\xi_s - \sigma)}f^D_s
\end{aligned}
$$

(2) If $s \in \{S^{\tilde{C}\tilde{D}}\}$

$$
\begin{aligned}
&\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^D_{s,t} \leq \theta_j \leq \bar{\theta}^I_{s,t}\Big] \\
&\quad= \mathbb{E}\Big[\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^D_{s,t} \leq \theta_j \leq \bar{\theta}^I_{s,t}, Y_t, G^E_{s,t}\Big]\Big] \quad \text{(by Law of Iterated Expectation)} \\
&\quad= \mathbb{E}\Big[\frac{1}{\Psi^D_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\big)^\sigma (G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^D_{s,t}}^{\bar{\theta}^I_{s,t}} \theta^\sigma \frac{h(\theta)}{H(\bar{\theta}^I_{s,t}) - H(\bar{\theta}^D_{s,t})} d\theta\Big] \\
&\quad= \mathbb{E}\Big[\frac{1}{\Psi^D_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\big)^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s \underline{\theta}^{\xi_s}}{\xi_s - \sigma}\frac{1}{\underline{\theta}^{\xi_s}}\frac{1}{(\bar{\theta}^D_{s,t})^{-\xi_s} - (\bar{\theta}^I_{s,t})^{-\xi_s}}\Big((\bar{\theta}^D_{s,t})^{\sigma-\xi_s} - (\bar{\theta}^I_{s,t})^{\sigma-\xi_s}\Big)\Big] \\
&\quad= \mathbb{E}\Big[\frac{1}{\Psi^D_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\big)^\sigma (G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s - \sigma}\frac{(\sigma-1)^{\frac{\sigma-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s-\sigma}(G^E_{s,t})^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}}{(\sigma-1)^{\frac{-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s}(G^E_{s,t})^{-\frac{\xi_s(\sigma-1)}{\sigma}}} \\
&\qquad\times \frac{\Big((f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^C_s - f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}\big(\frac{1}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\big)^{\frac{\sigma-\xi_s}{\sigma}}\Big)}{\Big((f^D_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^C_s - f^D_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}\big(\frac{1}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\big)^{\frac{-\xi_s}{\sigma}}\Big)}\Big] \\
&\quad= \mathbb{E}\Big[\frac{1}{\Psi^D_s}\frac{1}{\tilde{R}^D_s}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)\frac{(f^D_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}\Big(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^D_s)^{\frac{-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{-\xi_s}{\sigma}}\Big(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{-\xi_s}{\sigma}}}\Big] \\
&\quad= \mathbb{E}\Big[\frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{(f^D_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}\Big(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^D_s)^{\frac{-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{-\xi_s}{\sigma}}\Big(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{-\xi_s}{\sigma}}}\Big] \\
&\quad= \frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{(f^D_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{\sigma-\xi_s}{\sigma}}\Big(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^D_s)^{\frac{-\xi_s}{\sigma}} - (f^C_s - f^D_s)^{\frac{-\xi_s}{\sigma}}\Big(\frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{-\xi_s}{\sigma}}}
\end{aligned}
$$

(3) If $s \in \{S^{DC}, S^{\tilde{D}C}, S^{\tilde{D}\tilde{C}}\}$

$$
\begin{aligned}
&\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^I_{s,t} \le \theta_j\Big] \\
&\quad = \mathbb{E}\Big[\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^I_{s,t} \le \theta_j, Y_t, G^E_{s,t}\Big]\Big] \quad \text{(by Law of Iterated Expectation)} \\
&\quad = \mathbb{E}\Big[\frac{1}{\Psi^D_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\big)^\sigma (G^E_{s,t})^{1-\sigma} \int_{\bar{\theta}^I_{s,t}}^\infty \theta^\sigma \frac{h(\theta)}{H(\infty) - H(\bar{\theta}^I_{s,t})} d\theta\Big] \\
&\quad = \mathbb{E}\Big[\frac{1}{\Psi^D_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\big)^\sigma (G^E_{s,t})^{1-\sigma} \frac{\xi_s \underline{\theta}^{\xi_s}}{\xi_s - \sigma} \frac{1}{\underline{\theta}^{\xi_s}} \frac{1}{(\bar{\theta}^I_{s,t})^{-\xi_s}} \Big((\bar{\theta}^I_{s,t})^{\sigma - \xi_s}\Big)\Big] \\
&\quad = \mathbb{E}\Big[\frac{1}{\Psi^D_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^D_s}\big)^\sigma (G^E_{s,t})^{1-\sigma} \frac{\xi_s}{\xi_s - \sigma} \Big(\frac{((\sigma-1)(f^D_s - f^C_s))^{1/\sigma}}{\gamma\gamma_s Y_t} (G^E_{s,t}\tilde{R}^D_s\tilde{R}^C_s)^{\frac{\sigma-1}{\sigma}} \big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1} - (\tilde{R}^D_s)^{\sigma-1}}\big)^{\frac{1}{\sigma}}\Big)^\sigma\Big] \\
&\quad = \mathbb{E}\Big[\frac{1}{\Psi^D_s} \frac{1}{\tilde{R}^D_s} \frac{\xi_s}{\xi_s - \sigma} (\sigma - 1)(f^D_s - f^C_s) \frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1} - (\tilde{R}^D_s)^{\sigma-1}}\Big] \\
&\quad = \mathbb{E}\Big[\frac{\xi_s(\sigma-1)}{\Psi^D_s \tilde{R}^D_s (\xi_s - \sigma)} (f^D_s - f^C_s) \frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1} - (\tilde{R}^D_s)^{\sigma-1}}\Big] \\
&\quad = \frac{\xi_s(\sigma-1)}{\Psi^D_s \tilde{R}^D_s (\xi_s - \sigma)} (f^D_s - f^C_s) \frac{(\tilde{R}^C_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1} - (\tilde{R}^D_s)^{\sigma-1}}
\end{aligned}
$$

3.9.3 Chinese Loans

(1) If $s \in \{S^C, S^{\tilde{C}}\}$

$$
\begin{aligned}
\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^C_{s,t} \le \theta_j\Big] &= \mathbb{E}\Big[\mathbb{E}\Big[g^O_{s,j,t}\Big|\bar{\theta}^C_{s,t} \le \theta_j, Y_t, G^E_{s,t}\Big]\Big] \quad \text{(by Law of Iterated Expectation)} \\
&= \mathbb{E}\Big[\mathbb{E}\Big[\frac{1}{\Psi^C_s}\big(\frac{\theta_j \gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\Big|\bar{\theta}^C_{s,t} \le \theta_j, Y_t, G^E_{s,t}\Big]\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\mathbb{E}\Big[\theta^{\sigma}\Big|\bar{\theta}^C_{s,t} \le \theta_j, Y_t, G^E_{s,t}\Big]\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^C_{s,t}}^{\infty}\theta^{\sigma}\frac{h(\theta)}{H(\infty)-H(\bar{\theta}^C_{s,t})}d\theta\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\frac{\xi_s\underline{\theta}^{\xi_s}}{\xi_s-\sigma}\frac{(\bar{\theta}^C_{s,t})^{\xi_s}}{\underline{\theta}^{\xi_s}}\Big((\bar{\theta}^C_{s,t})^{\sigma-\xi_s}\Big)\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s-\sigma}\Big(\frac{\big((\sigma-1)f^C_s\big)^{1/\sigma}}{\gamma\gamma_s Y_t}(G^E_{s,t}\tilde{R}^C_s)^{\frac{\sigma-1}{\sigma}}\Big)^{\sigma}\Big] \\
&= \mathbb{E}\Big[\frac{1}{\Psi^C_s}\frac{1}{\tilde{R}^C_s}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)f^C_s\Big] \\
&= \mathbb{E}\Big[\frac{\xi_s(\sigma-1)}{\Psi^C_s\tilde{R}^C_s(\xi_s-\sigma)}f^C_s\Big] \\
&= \frac{\xi_s(\sigma-1)}{\Psi^C_s\tilde{R}^C_s(\xi_s-\sigma)}f^C_s
\end{aligned}
$$

(2) If $s \in \{S^{DC}, S^{\tilde{D}C}, S^{\tilde{D}\tilde{C}}\}$

$$
\begin{aligned}
&\mathbb{E}\left[g^O_{s,j,t}\middle|\bar{\theta}^C_{s,t} \leq \theta_j \leq \bar{\theta}^I_{s,t}\right] \\
&\quad = \mathbb{E}\left[\mathbb{E}\left[g^O_{s,j,t}\middle|\bar{\theta}^C_{s,t} \leq \theta_j \leq \bar{\theta}^I_{s,t}, Y_t, G^E_{s,t}\right]\right] \quad \text{(by Law of Iterated Expectation)} \\
&\quad = \mathbb{E}\left[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\int_{\bar{\theta}^C_{s,t}}^{\bar{\theta}^I_{s,t}} \theta^{\sigma}\frac{h(\theta)}{H(\bar{\theta}^I_{s,t}) - H(\bar{\theta}^C_{s,t})}d\theta\right] \\
&\quad = \mathbb{E}\left[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\frac{\xi_s\underline{\theta}^{\xi_s}}{\xi_s-\sigma}\frac{1}{\underline{\theta}^{\xi_s}}\frac{1}{(\bar{\theta}^C_{s,t})^{-\xi_s} - (\bar{\theta}^I_{s,t})^{-\xi_s}}\left((\bar{\theta}^C_{s,t})^{\sigma-\xi_s} - (\bar{\theta}^I_{s,t})^{\sigma-\xi_s}\right)\right] \\
&\quad = \mathbb{E}\Bigg[\frac{1}{\Psi^C_s}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^C_s}\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s-\sigma}\frac{(\sigma-1)^{\frac{\sigma-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s-\sigma}(G^E_{s,t})^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}}{(\sigma-1)^{\frac{-\xi_s}{\sigma}}(\gamma\gamma_s Y_t)^{\xi_s}(G^E_{s,t})^{-\frac{\xi_s(\sigma-1)}{\sigma}}} \\
&\qquad \times \frac{\Big((f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}} - (f^D_s - f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-\xi_s)(\sigma-1)}{\sigma}}\big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\big)^{\frac{\sigma-\xi_s}{\sigma}}\Big)}{\Big((f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}} - (f^D_s - f^C_s)^{\frac{-\xi_s}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{-\xi_s(\sigma-1)}{\sigma}}\big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\big)^{\frac{-\xi_s}{\sigma}}\Big)}\Bigg] \\
&\quad = \mathbb{E}\left[\frac{1}{\Psi^C_s}\frac{1}{\tilde{R}^C_s}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)\frac{(f^C_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^C_s)^{\frac{-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{-\xi_s}{\sigma}}}\right] \\
&\quad = \mathbb{E}\left[\frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{(f^C_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^C_s)^{\frac{-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{-\xi_s}{\sigma}}}\right] \\
&\quad = \frac{\xi_s(\sigma-1)}{\Psi^D_s\tilde{R}^D_s(\xi_s-\sigma)}\frac{(f^C_s)^{\frac{\sigma-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{\sigma-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{\sigma-\xi_s}{\sigma}}}{(f^C_s)^{\frac{-\xi_s}{\sigma}} - (f^D_s - f^C_s)^{\frac{-\xi_s}{\sigma}}\left(\frac{(\tilde{R}^D_s)^{\sigma-1}}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\right)^{\frac{-\xi_s}{\sigma}}}
\end{aligned}
$$

(3) If $s \in \{S^{\tilde{C}\tilde{D}}\}$

$$
\begin{aligned}
&\mathbb{E}\bigg[g^{O}_{s,j,t}\bigg|\bar{\theta}^{I}_{s,t} \le \theta_j\bigg] \\
&= \mathbb{E}\bigg[\mathbb{E}\bigg[g^{O}_{s,j,t}\bigg|\bar{\theta}^{I}_{s,t} \le \theta_j, Y_t, G^{E}_{s,t}\bigg]\bigg] \quad \text{(by Law of Iterated Expectation)} \\
&= \mathbb{E}\bigg[\frac{1}{\Psi^{C}_{s}}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^{C}_{s}}\big)^{\sigma}(G^{E}_{s,t})^{1-\sigma}\int_{\bar{\theta}^{I}_{s,t}}^{\infty}\theta^{\sigma}\frac{h(\theta)}{H(\infty)-H(\bar{\theta}^{I}_{s,t})}d\theta\bigg] \\
&= \mathbb{E}\bigg[\frac{1}{\Psi^{C}_{s}}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^{C}_{s}}\big)^{\sigma}(G^{E}_{s,t})^{1-\sigma}\frac{\xi_s\underline{\theta}^{\xi_s}}{\xi_s-\sigma}\frac{1}{\underline{\theta}^{\xi_s}}\frac{1}{(\bar{\theta}^{I}_{s,t})^{-\xi_s}}\bigg((\bar{\theta}^{I}_{s,t})^{\sigma-\xi_s}\bigg)\bigg] \\
&= \mathbb{E}\bigg[\frac{1}{\Psi^{C}_{s}}\big(\frac{\gamma\gamma_s Y_t}{\tilde{R}^{C}_{s}}\big)^{\sigma}(G^{E}_{s,t})^{1-\sigma}\frac{\xi_s}{\xi_s-\sigma}\bigg(\frac{((\sigma-1)(f^{C}_{s}-f^{D}_{s}))^{1/\sigma}}{\gamma\gamma_s Y_t}(G^{E}_{s,t}\tilde{R}^{D}_{s}\tilde{R}^{C}_{s})^{\frac{\sigma-1}{\sigma}}\big(\frac{1}{(\tilde{R}^{D}_{s})^{\sigma-1}-(\tilde{R}^{C}_{s})^{\sigma-1}}\big)^{\frac{1}{\sigma}}\bigg)^{\sigma}\bigg] \\
&= \mathbb{E}\bigg[\frac{1}{\Psi^{C}_{s}}\frac{1}{\tilde{R}^{C}_{s}}\frac{\xi_s}{\xi_s-\sigma}(\sigma-1)(f^{C}_{s}-f^{D}_{s})\frac{(\tilde{R}^{D}_{s})^{\sigma-1}}{(\tilde{R}^{D}_{s})^{\sigma-1}-(\tilde{R}^{C}_{s})^{\sigma-1}}\bigg] \\
&= \mathbb{E}\bigg[\frac{\xi_s(\sigma-1)}{\Psi^{C}_{s}\tilde{R}^{C}_{s}(\xi_s-\sigma)}(f^{C}_{s}-f^{D}_{s})\frac{(\tilde{R}^{D}_{s})^{\sigma-1}}{(\tilde{R}^{D}_{s})^{\sigma-1}-(\tilde{R}^{C}_{s})^{\sigma-1}}\bigg] \\
&= \frac{\xi_s(\sigma-1)}{\Psi^{C}_{s}\tilde{R}^{C}_{s}(\xi_s-\sigma)}(f^{C}_{s}-f^{D}_{s})\frac{(\tilde{R}^{D}_{s})^{\sigma-1}}{(\tilde{R}^{D}_{s})^{\sigma-1}-(\tilde{R}^{C}_{s})^{\sigma-1}}
\end{aligned}
$$

$\square$

3.10 Extension of Proposition 3 and Its Proof

(Extended Aggregation of the Sectoral Effective Public Capital). The effective public capital in sector s for period t is given by:

$$G^E_{s,t} = \mathcal{G}^E_s \cdot Y_t^{\frac{\sigma(\xi-1)}{\xi(\sigma-1)}},$$

where

$$\mathcal{G}^E_s = \begin{cases} \mathcal{G}^{E,D}_s \cdot \mathcal{G}_s & \text{if } s \in (S^{DG} \cup S^{\tilde{D}\tilde{G}} \cup S^G \cup S^{\tilde{G}}) \\ \mathcal{G}^{E,C}_s \cdot \mathcal{G}_s & \text{if } s \in S^{\tilde{C}\tilde{G}} \\ \mathcal{G}^{E,DC}_s \cdot \mathcal{G}_s & \text{if } s \in (S^{DCG} \cup S^{\tilde{D}C\tilde{G}} \cup S^{\tilde{D}\tilde{C}\tilde{G}}) \\ \mathcal{G}^{E,CD}_s \cdot \mathcal{G}_s & \text{if } s \in S^{\tilde{C}\tilde{D}\tilde{G}}. \end{cases}$$

Here, $\mathcal{G}_s$ is a factor not related to the financing choices, defined as:

$$\mathcal{G}_s \equiv \big(\sigma - 1\big)^{\frac{\sigma-\xi}{\xi(\sigma-1)}} \big(\gamma\gamma_s\big)^{\frac{\sigma(\xi-1)}{\xi(\sigma-1)}} \left(\frac{\xi {\theta_{min}}^{\xi}}{\xi - \sigma}\right)^{\frac{\sigma}{\xi(\sigma-1)}}$$

and the other financing-specific factors are:

$$\begin{aligned}
\mathcal{G}^{E,D}_s &\equiv (\tilde{R}^D_s)^{-1}(\tilde{f}^G_s)^{\frac{\sigma-\xi}{\xi(\sigma-1)}} \\
\mathcal{G}^{E,C}_s &\equiv \left[(\tilde{f}^G_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(1-\sigma)\xi}{\sigma}} + (1 - (\frac{\tilde{R}^C_s}{\tilde{R}^D_s})^{\sigma-1})(f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^C_s)^{\frac{(1-\sigma)\xi}{\sigma}}\right]^{\frac{\sigma}{\xi(\sigma-1)}} \\
\mathcal{G}^{E,DC}_s &\equiv \Bigg[((\tilde{R}^D_s)^{1-\sigma} - (\tilde{R}^C_s)^{1-\sigma})^{\frac{\xi}{\sigma}}(f^D_s - f^C_s)^{\frac{\sigma-\xi}{\sigma}} + (\tilde{f}^G_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(1-\sigma)\xi}{\sigma}} \\
&\qquad + (1 - (\frac{\tilde{R}^C_s}{\tilde{R}^D_s})^{\sigma-1})(f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^C_s)^{\frac{(1-\sigma)\xi}{\sigma}}\Bigg]^{\frac{\sigma}{\xi(\sigma-1)}} \\
\mathcal{G}^{E,CD}_s &\equiv \left[((\tilde{R}^C_s)^{1-\sigma} - (\tilde{R}^D_s)^{1-\sigma})^{\frac{\xi}{\sigma}}(f^C_s - f^D_s)^{\frac{\sigma-\xi}{\sigma}} + (\tilde{f}^G_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(1-\sigma)\xi}{\sigma}}\right]^{\frac{\sigma}{\xi(\sigma-1)}}.
\end{aligned}$$

The total effective public capital stock in s in t, $G^E_{s,t}$, is

$$\begin{aligned}
\left[\int \theta \cdot (g^E_{s,t}(\theta))^{\frac{\sigma-1}{\sigma}} dH(\theta)\right]^{\frac{\sigma}{\sigma-1}} &= \left[\int \theta \cdot (\frac{\theta\gamma\gamma_s Y_t}{\tilde{R}^p})^{\sigma-1}(G^E_{s,t})^{\frac{(1-\sigma)(\sigma-1)}{\sigma}} \frac{\xi\theta^{\xi}_{min}}{\theta^{\xi+1}} d\theta\right]^{\frac{\sigma}{\sigma-1}} \\
&= (\gamma\gamma_s Y_t)^{\sigma}(G^E_{s,t})^{1-\sigma}(\xi\theta^{\xi}_{min})^{\frac{\sigma}{\sigma-1}}\left[\int (\tilde{R}^p)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

If $s \in \{S^G, S^{\tilde{G}}\}$,

$$\left[\int (\tilde{R}_s^p)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}} = \left[\int_{\bar{\theta}_{s,t}^G}^{\infty}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}}$$
$$= \left[(\tilde{R}_s^D)^{1-\sigma}\frac{1}{\xi-\sigma}(\bar{\theta}_{s,t}^G)^{\sigma-\xi}\right]^{\frac{\sigma}{\sigma-1}}$$
$$= (\tilde{R}^D)^{-\sigma}(\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{\left((\sigma-1)\tilde{f}_s^G\right)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}(G_{s,t}^E\tilde{R}_s^D)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}$$
$$= (\tilde{R}^D)^{-\xi}(\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{\left((\sigma-1)\tilde{f}_s^G\right)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$

If $s \in \{S^{DG}, S^{\tilde{D}\tilde{G}}\}$,

$$\left[\int (\tilde{R}_s^p)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}} = \left[\int_{\bar{\theta}_{s,t}^D}^{\infty}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^G}^{\bar{\theta}_{s,t}^D}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}}$$
$$= \left[\int_{\bar{\theta}_{s,t}^G}^{\infty}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}}$$
$$= (\tilde{R}^D)^{-\xi}(\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{\left((\sigma-1)\tilde{f}_s^G\right)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$

If $s \in \{S^{\tilde{C}\tilde{G}}\}$,

$$\left[\int (\tilde{R}_s^p)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}} = \left[\int_{\bar{\theta}_{s,t}^C}^{\infty}(\tilde{R}_s^C)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^G}^{\bar{\theta}_{s,t}^C}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}}$$
$$= \left[(\tilde{R}_s^C)^{1-\sigma}\frac{1}{\xi-\sigma}(\bar{\theta}_s^C)^{\sigma-\xi} + (\tilde{R}_s^D)^{1-\sigma}\frac{1}{\xi-\sigma}((\bar{\theta}_{s,t}^G)^{\sigma-\xi} - (\bar{\theta}_{s,t}^C)^{\sigma-\xi})\right]^{\frac{\sigma}{\sigma-1}}$$
$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[(\tilde{R}_s^D)^{1-\sigma}(\bar{\theta}_{s,t}^G)^{\sigma-\xi} + ((\tilde{R}_s^C)^{1-\sigma} - (\tilde{R}_s^D)^{1-\sigma})(\bar{\theta}_{s,t}^C)^{\sigma-\xi}\right]^{\frac{\sigma}{\sigma-1}}$$
$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$
$$\cdot\left[(\tilde{R}_s^D)^{1-\sigma}(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}} + ((\tilde{R}_s^C)^{1-\sigma} - (\tilde{R}_s^D)^{1-\sigma})(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$
$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$
$$\cdot\left[(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(1-\sigma)\xi}{\sigma}} + (1 - (\frac{\tilde{R}_s^C}{\tilde{R}_s^D})^{\sigma-1})(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(1-\sigma)\xi}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

If $s \in \{S^{DCG}, S^{\tilde{D}\tilde{C}\tilde{G}}\}$,

$$\Big[\int (\tilde{R}_s^p)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\Big]^{\frac{\sigma}{\sigma-1}}$$

$$= \Big[\int_{\bar{\theta}_{s,t}^I}^{\infty}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^C}^{\bar{\theta}_{s,t}^I}(\tilde{R}_s^C)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^G}^{\bar{\theta}_{s,t}^C}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\Big]^{\frac{\sigma}{\sigma-1}}$$

$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\Big[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\Big]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$
$$\cdot\Big[(\tilde{R}_s^D)^{1-\sigma}(f_s^D-f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}(\frac{1}{(\tilde{R}_s^C)^{\sigma-1}-(\tilde{R}_s^D)^{\sigma-1}})^{\frac{\sigma-\xi}{\sigma}}$$
$$-(\tilde{R}_s^C)^{1-\sigma}(f_s^D-f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}(\frac{1}{(\tilde{R}_s^C)^{\sigma-1}-(\tilde{R}_s^D)^{\sigma-1}})^{\frac{\sigma-\xi}{\sigma}}$$
$$+(\tilde{R}_s^C)^{1-\sigma}(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}-(\tilde{R}_s^D)^{1-\sigma}(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}$$
$$+(\tilde{R}_s^D)^{1-\sigma}(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}}$$

$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\Big[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\Big]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$
$$\cdot\Big[((\tilde{R}_s^D)^{1-\sigma}-(\tilde{R}_s^C)^{1-\sigma})(f_s^D-f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}(\frac{(\tilde{R}_s^D\tilde{R}_s^C)^{1-\sigma}}{(\tilde{R}_s^D)^{1-\sigma}-(\tilde{R}_s^C)^{1-\sigma}})^{\frac{\sigma-\xi}{\sigma}}$$
$$+((\tilde{R}_s^C)^{1-\sigma}-(\tilde{R}_s^D)^{1-\sigma})(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}$$
$$+(\tilde{R}_s^D)^{1-\sigma}(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}}$$

$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\Big[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\Big]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$
$$\cdot\Big[((\tilde{R}_s^D)^{1-\sigma}-(\tilde{R}_s^C)^{1-\sigma})^{\frac{\xi}{\sigma}}(f_s^D-f_s^C)^{\frac{\sigma-\xi}{\sigma}}+((\tilde{R}_s^C)^{1-\sigma}-(\tilde{R}_s^D)^{1-\sigma})(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}$$
$$+(\tilde{R}_s^D)^{1-\sigma}(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}}$$

$$= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\Big[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\Big]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$$
$$\cdot\Big[((\tilde{R}_s^D)^{1-\sigma}-(\tilde{R}_s^C)^{1-\sigma})^{\frac{\xi}{\sigma}}(f_s^D-f_s^C)^{\frac{\sigma-\xi}{\sigma}}+(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(1-\sigma)\xi}{\sigma}}$$
$$+(1-(\frac{\tilde{R}_s^C}{\tilde{R}_s^D})^{\sigma-1})(f_s^C)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^C)^{\frac{(1-\sigma)\xi}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}}$$

Lastly, if $s \in \{S^{\tilde{C}\tilde{D}\tilde{G}}\}$,

$$
\begin{aligned}
&\left[\int (\tilde{R}_s^p)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}} \\
&\quad= \left[\int_{\bar{\theta}_{s,t}^I}^{\infty}(\tilde{R}_s^C)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^D}^{\bar{\theta}_{s,t}^I}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^G}^{\bar{\theta}_{s,t}^D}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}} \\
&\quad= \left[\int_{\bar{\theta}_{s,t}^I}^{\infty}(\tilde{R}_s^C)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta + \int_{\bar{\theta}_{s,t}^G}^{\bar{\theta}_{s,t}^I}(\tilde{R}_s^D)^{1-\sigma}\theta^{\sigma-\xi-1}d\theta\right]^{\frac{\sigma}{\sigma-1}} \\
&\quad= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi} \\
&\qquad\cdot\Bigg[(\tilde{R}_s^C)^{1-\sigma}(f_s^C-f_s^D)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}(\frac{1}{(\tilde{R}_s^D)^{\sigma-1}-(\tilde{R}_s^C)^{\sigma-1}})^{\frac{\sigma-\xi}{\sigma}} \\
&\qquad-(\tilde{R}_s^D)^{1-\sigma}(f_s^C-f_s^D)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D\tilde{R}_s^C)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}(\frac{1}{(\tilde{R}_s^D)^{\sigma-1}-(\tilde{R}_s^C)^{\sigma-1}})^{\frac{\sigma-\xi}{\sigma}} \\
&\qquad+(\tilde{R}_s^D)^{1-\sigma}(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Bigg]^{\frac{\sigma}{\sigma-1}} \\
&\quad= (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi} \\
&\qquad\cdot\left[((\tilde{R}_s^C)^{1-\sigma}-(\tilde{R}_s^D)^{1-\sigma})^{\frac{\xi}{\sigma}}(f_s^C-f_s^D)^{\frac{\sigma-\xi}{\sigma}}+(\tilde{f}_s^G)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}_s^D)^{\frac{(1-\sigma)\xi}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Note that $(\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi}$ is an additional common factor invariant to sectoral financing. Then, the common factor is

$$
\begin{aligned}
&(\gamma\gamma_s Y_t)^{\sigma}(G_{s,t}^E)^{1-\sigma}(\xi\theta_{min}^{\xi})^{\frac{\sigma}{\sigma-1}} \times (\frac{1}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}\left[\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}\right]^{\frac{\sigma(\sigma-\xi)}{\sigma-1}}(G_{s,t}^E)^{\sigma-\xi} \\
&= (\frac{\xi\theta_{min}^{\xi}}{\xi-\sigma})^{\frac{\sigma}{\sigma-1}}(\sigma-1)^{\frac{\sigma-\xi}{\sigma-1}}\big(\gamma\gamma_s Y_t\big)^{\frac{\sigma(\xi-1)}{\sigma-1}}(G_{s,t}^E)^{1-\xi}
\end{aligned}
$$

Hence,

$$
G_{s,t}^E = (\frac{\xi\theta_{min}^{\xi}}{\xi-\sigma})^{\frac{\sigma}{\xi(\sigma-1)}}(\sigma-1)^{\frac{\sigma-\xi}{\xi(\sigma-1)}}\big(\gamma\gamma_s Y_t\big)^{\frac{\sigma(\xi-1)}{\xi(\sigma-1)}}\mathcal{G}_s^E
$$

4 Additional Theoretical Results

4.1 Effective Public Capital vs Observed Public Capital

Here, I show how the observed public capital in the data can be theoretically expressed in terms of model parameters and GDP.

$$
\begin{aligned}
g^E_{s,j,t} &= (\frac{\theta_j \gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} Y_t^{\sigma_s} (G^E_{st})^{1-\sigma_s} \\
&= (\frac{\theta_j \gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}}
\end{aligned}
$$

Let $g^O_{s,j,t}$ denote the observed size of project j. Then, $g^O_{s,j,t} = o_{s,j} g^E_{s,j,t}$ where $o_{s,j}$ takes the value of 1 if j is not diverted and $1/\psi^p_s$ if it is maximally diverted. Then,

$$
\begin{aligned}
G^O_{s,j,t} &= \int o_{s,j} g^E_{s,j,t} dj \\
&= \int o_{s,j} g^E_{s,j,t} dj \\
&= \int o_{s,j} (\frac{\theta_j \gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} dj \\
&= \int o_{s,j} (\frac{\theta \gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} h(\theta) d\theta \\
&= o_s (\frac{\gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \xi_s {\theta^s_{min}}^{\xi_s} \int \theta^{\sigma_s - \xi_s - 1} d\theta \\
&= o_s (\frac{\gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \xi_s {\theta^s_{min}}^{\xi_s} \frac{1}{\xi_s - \sigma_s} (\bar{\theta}^p_{s,t})^{\sigma_s - \xi_s} \\
&= o_s (\frac{\gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \xi_s {\theta^s_{min}}^{\xi_s} \frac{1}{\xi_s - \sigma_s} (\frac{((\sigma_s - 1) f^p_s)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t} (\mathcal{G}^E_s \tilde{R}^p_s Y_t^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}})^{\frac{\sigma_s - 1}{\sigma_s}})^{\sigma_s - \xi_s} \\
&= o_s (\gamma\gamma_s)^{\xi_s} (\tilde{R}^p_s)^{\frac{-\sigma_s - \xi_s\sigma_s + \xi_s}{\sigma_s}} (\mathcal{G}^E_s)^{\frac{-\xi_s(\sigma_s - 1)}{\sigma_s}} \frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s - \sigma_s} ((\sigma_s - 1) f^p_s)^{\frac{\sigma_s - \xi_s}{\sigma_s}} Y_t \\
&= o_s (\mathcal{G}^E_s)^{\frac{-\xi_s(\sigma_s - 1)}{\sigma_s}} (\gamma\gamma_s)(\tilde{R}^p_s)^{-1} \left((\gamma\gamma_s)^{\xi_s - 1} (\tilde{R}^p_s)^{\frac{-\xi_s(\sigma_s - 1)}{\sigma_s}} ((\sigma_s - 1) f^p_s)^{\frac{\sigma_s - \xi_s}{\sigma_s}} \frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s - \sigma_s} \right) Y_t \\
&= o_s (\mathcal{G}^E_s)^{\frac{-\xi_s(\sigma_s - 1)}{\sigma_s}} (\gamma\gamma_s)(\tilde{R}^p_s)^{-1} (\mathcal{G}^E_s)^{\frac{\xi_s(\sigma_s - 1)}{\sigma_s}} Y_t \\
&= o_s \frac{\gamma\gamma_s}{\tilde{R}^p_s} Y_t
\end{aligned}
$$

Meanwhile, $G^E_{s,t} = \mathcal{G}^E_s Y_t^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}}$. Then,

$$\begin{aligned} G^O_{s,t} &= o_s \frac{\gamma\gamma_s}{\tilde{R}^p_s} \left(\frac{\mathcal{G}^E_s}{\mathcal{G}^E_s} Y_t^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} \right)^{\frac{\xi_s(\sigma_s-1)}{\sigma_s(\xi_s-1)}} \\ &= o_s \frac{\gamma\gamma_s}{\tilde{R}^p_s} (\mathcal{G}^E_s)^{-\frac{\xi_s(\sigma_s-1)}{\sigma_s(\xi_s-1)}} (G^E_{s,t})^{\frac{\xi_s(\sigma_s-1)}{\sigma_s(\xi_s-1)}} \end{aligned}$$

Rearranging,

$$\begin{aligned} G^E_{s,t} &= \left[\frac{\tilde{R}^p_s}{o_s\gamma\gamma_s} (\mathcal{G}^E_s)^{\frac{\xi_s(\sigma_s-1)}{\sigma_s(\xi_s-1)}} G^O_{s,t} \right]^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} \\ &= \mathcal{G}^E_s \left(\frac{\tilde{R}^p_s}{o_s\gamma\gamma_s} \right)^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} (G^O_{s,t})^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} \end{aligned}$$

Then,

$$\begin{aligned} g^E_{s,j,t} &= (\frac{\theta_j\gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} Y_t^{\sigma_s} (G^E_{st})^{1-\sigma_s} \\ &= (\frac{\theta_j\gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} Y_t^{\sigma_s} \left(\mathcal{G}^E_s \left(\frac{\tilde{R}^p_s}{o_s\gamma\gamma_s} \right)^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} (G^O_{s,t})^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} \right)^{1-\sigma_s} \\ &= \theta_j^{\sigma_s} (\frac{\gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s + \frac{\sigma_s(\xi_s-1)}{\xi_s}} o_s^{\frac{\sigma_s(\xi_s-1)}{\xi_s}} (\mathcal{G}^E_s)^{1-\sigma_s} (G^O_{s,t})^{-\frac{\sigma_s(\xi_s-1)}{\xi_s}} Y_t^{\sigma_s} \end{aligned}$$

Now suppose a sector that is financed by both providers.

$$\begin{aligned}
G^O_{s,j,t} &= \int o_{s,j} g^E_{s,j,t} dj \\
&= \int o_{s,j} g^E_{s,j,t} dj \\
&= \int o_{s,j} (\frac{\theta_j \gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} dj \\
&= \int o_{s,j} (\frac{\theta \gamma\gamma_s}{\tilde{R}^p_s})^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} h(\theta) d\theta \\
&= (\gamma\gamma_s)^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \xi_s {\theta^s_{min}}^{\xi_s} \left(o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s} \int_{\bar{\theta}^{p'}_{s,t}}^{\bar{\theta}^I_{s,t}} \theta^{\sigma_s - \xi_s - 1} d\theta + o^p_s (\tilde{R}^p_s)^{-\sigma_s} \int_{\bar{\theta}^I_{s,t}}^{\infty} \theta^{\sigma_s - \xi_s - 1} d\theta \right) \\
&= (\gamma\gamma_s)^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \xi_s {\theta^s_{min}}^{\xi_s} \left[\frac{o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s}}{\sigma_s - \xi_s} \theta_s^{\sigma_s - \xi_s} \Big|_{\bar{\theta}^{p'}_{s,t+1}}^{\bar{\theta}^I_{s,t+1}} + \frac{o^p_s (\tilde{R}^p_s)^{-\sigma_s}}{\sigma_s - \xi_s} \theta_s^{\sigma_s - \xi_s} \Big|_{\bar{\theta}^I_{s,t+1}}^{\infty} \right] \\
&= (\gamma\gamma_s)^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \xi_s {\theta^s_{min}}^{\xi_s} \left[\frac{o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s}}{\sigma_s - \xi_s} ((\bar{\theta}^I_{s,t+1})^{\sigma_s - \xi_s} - (\bar{\theta}^{p'}_{s,t+1})^{\sigma_s - \xi_s}) - \frac{o^p_s (\tilde{R}^p_s)^{-\sigma_s}}{\sigma_s - \xi_s} (\bar{\theta}^I_{s,t+1})^{\sigma_s - \xi_s} \right] \\
&= (\gamma\gamma_s)^{\sigma_s} (\mathcal{G}^E_s)^{1-\sigma_s} Y_t^{\frac{\sigma_s}{\xi_s}} \frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s - \sigma_s} \left[(o^p_s (\tilde{R}^p_s)^{-\sigma_s} - o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s}) (\bar{\theta}^I_{s,t+1})^{\sigma_s - \xi_s} + o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s} (\bar{\theta}^{p'}_{s,t+1})^{\sigma_s - \xi_s} \right] \\
&= \left[(o^p_s (\tilde{R}^p_s)^{-\sigma_s} - o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s}) \big(\frac{f^p_s - f^{p'}_s}{(\tilde{R}^p_s)^{1-\sigma_s} - (\tilde{R}^{p'}_s)^{1-\sigma_s}} \big)^{\frac{\sigma_s - \xi_s}{\sigma_s}} + o^{p'}_s (\tilde{R}^{p'}_s)^{-\sigma_s} (\frac{f^{p'}_s}{(\tilde{R}^{p'}_s)^{1-\sigma_s}})^{\frac{\sigma_s - \xi_s}{\sigma_s}} \right] \\
&\quad \times (\gamma\gamma_s)^{\xi_s} (\mathcal{G}^E_s)^{-\frac{\xi_s(\sigma_s - 1)}{\sigma_s}} \frac{\xi_s {\theta^s_{min}}^{\xi_s}}{\xi_s - \sigma_s} (\sigma_s - 1)^{\frac{\sigma_s - \xi_s}{\sigma_s}} Y_t \\
&= \mathcal{Y} Y_t
\end{aligned}$$

Meanwhile, $G^E_{s,t} = \mathcal{G}^E_s Y_t^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}}$. Then,

$$\begin{aligned}
G^O_{s,t} &= \mathcal{Y} \left(\frac{\mathcal{G}^E_s}{\mathcal{G}^E_s} Y_t^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}} \right)^{\frac{\xi_s(\sigma_s - 1)}{\sigma_s(\xi_s - 1)}} \\
& \mathcal{Y} (\mathcal{G}^E_s)^{-\frac{\xi_s(\sigma_s - 1)}{\sigma_s(\xi_s - 1)}} (G^E_{s,t})^{\frac{\xi_s(\sigma_s - 1)}{\sigma_s(\xi_s - 1)}}
\end{aligned}$$

Rearranging,

$$\begin{aligned}
G^E_{s,t} &= \left[\frac{1}{\mathcal{Y}} (\mathcal{G}^E_s)^{\frac{\xi_s(\sigma_s - 1)}{\sigma_s(\xi_s - 1)}} G^O_{s,t} \right]^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}} \\
&= \mathcal{G}^E_s \left(\frac{1}{\mathcal{Y}} \right)^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}} (G^O_{s,t})^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}}
\end{aligned}$$

Then,

$$\begin{aligned}
g^E_{s,j,t} &= (\frac{\theta_j \gamma \gamma_s}{\tilde{R}^p_s})^{\sigma_s} Y_t^{\sigma_s} (G^E_{st})^{1-\sigma_s} \\
&= (\frac{\theta_j \gamma \gamma_s}{\tilde{R}^p_s})^{\sigma_s} Y_t^{\sigma_s} \left(\mathcal{G}^E_s \left(\frac{1}{\mathcal{Y}} \right)^{\frac{\sigma_s(\xi_s - 1)}{\xi_s(\sigma_s - 1)}} (G^O_{s,t})^{\frac{\sigma_s(\xi_s-1)}{\xi_s(\sigma_s-1)}} \right)^{1-\sigma_s}
\end{aligned}$$

Hence, in any case,

$$\begin{aligned}
g^O_{s,j,t} &= o_s g^E_{s,j,t} \\
&= \theta_j^{\sigma_s} \mathcal{A}_s (G^O_{s,t})^{-\frac{\sigma_s(\xi_s-1)}{\xi_s}} Y_t^{\sigma_s}
\end{aligned}$$

with $\mathcal{A}_s$ being some constant.

4.2 Debt Stock to GDP

(Debt Stock to GDP Ratio). The ratio of debt stock owed to p in sector s in period t to GDP is given by:

$$\frac{D^p_{s,t}}{Y_t} = \frac{\gamma\gamma_s}{\Psi^p_s}\frac{1}{(\tilde{R}^p_s)^\sigma}\mathcal{D}^p_s,$$

and

$$\mathcal{D}^p_s = \begin{cases} (\mathcal{G}^{E,D}_s)^{\frac{\xi(1-\sigma)}{\sigma}}\mathcal{D}^{p,D}_s & \text{if } s \in (S^{DG} \cup S^{\tilde{D}\tilde{G}} \cup S^{G} \cup S^{\tilde{G}}) \\ (\mathcal{G}^{E,C}_s)^{\frac{\xi(1-\sigma)}{\sigma}}\mathcal{D}^{p,C}_s & \text{if } s \in S^{\tilde{C}\tilde{G}} \\ (\mathcal{G}^{E,DC})^{\frac{\xi(1-\sigma)}{\sigma}}\mathcal{D}^{p,DC}_s & \text{if } s \in (S^{DCG} \cup S^{\tilde{D}C\tilde{G}} \cup S^{\tilde{D}\tilde{C}\tilde{G}}) \\ (\mathcal{G}^{E,CD})^{\frac{\xi(1-\sigma)}{\sigma}}\mathcal{D}^{p,CD}_s & \text{if } s \in S^{\tilde{C}\tilde{D}\tilde{G}}. \end{cases}$$

where

$$\mathcal{D}^{D,D}_s \equiv (f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}$$

$$\mathcal{D}^{D,C}_s \equiv 0$$

$$\mathcal{D}^{D,DC}_s \equiv \left[(f^D_s - f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1} - (\tilde{R}^D_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi}{\sigma}}\right]$$

$$\mathcal{D}^{D,CD}_s \equiv \left[(f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}} - (f^C_s - f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big(\frac{1}{(\tilde{R}^D_s)^{\sigma-1} - (\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi}{\sigma}}\right]$$

$$\mathcal{D}^{C,D}_s \equiv 0$$

$$\mathcal{D}^{C,C}_s \equiv (f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}$$

$$\mathcal{D}^{C,DC}_s \equiv \left[(f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}} - (f^D_s - f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1} - (\tilde{R}^D_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi}{\sigma}}\right]$$

$$\mathcal{D}^{C,CD}_s \equiv \left[(f^C_s - f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\Big(\frac{1}{(\tilde{R}^D_s)^{\sigma-1} - (\tilde{R}^C_s)^{\sigma-1}}\Big)^{\frac{\sigma-\xi}{\sigma}}\right]$$

First, note that $d^p_{sjt} > 0$ only for projects with productivity $\theta \in [\underline{\theta}, \bar{\theta})$ for some $\underline{\theta}$ and $\bar{\theta}$. Then,

$$\begin{aligned} D^p_{st} &= \int d^p_{sjt}dj \\ &= \int \frac{1}{\Psi^p_s}g^e_{sjt}dj \\ &= \int \frac{1}{\Psi^p_s}\Big(\frac{\theta\gamma\gamma_s}{\tilde{R}^p_s}Y_t\Big)^\sigma(G^E_{s,t})^{1-\sigma}\frac{\xi\theta^\xi_{min}}{\theta^{\xi+1}}d\theta \\ &= \frac{1}{\Psi^p_s}\Big(\frac{\theta\gamma\gamma_s}{\tilde{R}^p_s}Y_t\Big)^\sigma(G^E_{s,t})^{1-\sigma}\frac{\xi\theta^\xi_{min}}{\xi-\sigma}(\underline{\theta}^{\sigma-\xi} - \bar{\theta}^{\sigma-\xi}) \end{aligned}$$

Note that the thresholds $\underline{\theta}$ and $\bar{\theta}$ are either the zero-profit cutoff or financing indifference cutoff. All those cutoffs have $\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}(G^E_{st})^{\frac{\sigma-1}{\sigma}}$ as a common factor. Let's denote the remaining factors of $\underline{\theta}$ and $\bar{\theta}$ by $\underline{\theta}_{resid}$ and $\bar{\theta}_{resid}$ respectively. Then,

$$\begin{aligned}
D^p_{st} &= \frac{1}{\Psi^p_s}\big(\frac{\theta\gamma\gamma_s}{\tilde{R}^p_s}Y_t\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\frac{\xi\theta^{\xi}_{min}}{\xi-\sigma}(\underline{\theta}^{\sigma-\xi}-\bar{\theta}^{\sigma-\xi}) \\
&= frac1\Psi^p_s\big(\frac{\theta\gamma\gamma_s}{\tilde{R}^p_s}Y_t\big)^{\sigma}(G^E_{s,t})^{1-\sigma}\frac{\xi\theta^{\xi}_{min}}{\xi-\sigma}\big(\frac{(\sigma-1)^{\frac{1}{\sigma}}}{\gamma\gamma_s Y_t}(G^E_{st})^{\frac{\sigma-1}{\sigma}}\big)^{\sigma-\xi}(\underline{\theta}^{\sigma-\xi}_{resid}-\bar{\theta}^{\sigma-\xi}_{resid}) \\
&= \frac{1}{\Psi^p_s}(\sigma-1)^{\frac{\sigma-\xi}{\sigma}}(\gamma\gamma_s Y_t)^{\xi}(G^E_{st})^{\frac{\xi(1-\sigma)}{\sigma}}(\tilde{R}^p_s)^{-\sigma}\frac{\xi\theta^{\xi}_{min}}{\xi-\sigma}(\underline{\theta}^{\sigma-\xi}_{resid}-\bar{\theta}^{\sigma-\xi}_{resid}) \\
&= \frac{1}{\Psi^p_s}(\sigma-1)^{\frac{\sigma-\xi}{\sigma}}(\gamma\gamma_s Y_t)^{\xi}(\mathcal{G}_s\mathcal{G}^{E,f}_s Y_t^{\frac{\sigma(\xi-1)}{\xi(\sigma-1)}})^{\frac{\xi(1-\sigma)}{\sigma}}(\tilde{R}^p_s)^{-\sigma}\frac{\xi\theta^{\xi}_{min}}{\xi-\sigma}(\underline{\theta}^{\sigma-\xi}_{resid}-\bar{\theta}^{\sigma-\xi}_{resid}) \\
&= \frac{1}{\Psi^p_s}\gamma\gamma_s Y_t(\tilde{R}^p_s)^{-\sigma}(\mathcal{G}^{E,f}_s)^{\frac{\xi(1-\sigma)}{\sigma}}(\underline{\theta}^{\sigma-\xi}_{resid}-\bar{\theta}^{\sigma-\xi}_{resid})
\end{aligned}$$

Let $\mathcal{D}^{p,f}$ denote $(\underline{\theta}^{\sigma-\xi}_{resid}-\bar{\theta}^{\sigma-\xi}_{resid})$ for each donor p and financing mode f. Then,

$$\begin{aligned}
\mathcal{D}^{D,D}_s &\equiv (f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}} \\
\mathcal{D}^{D,C}_s &\equiv 0 \\
\mathcal{D}^{D,DC}_s &\equiv \Big[(f^D_s-f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\big)^{\frac{\sigma-\xi}{\sigma}}\Big] \\
\mathcal{D}^{D,CD}_s &\equiv \Big[(f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}-(f^C_s-f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\big(\frac{1}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\big)^{\frac{\sigma-\xi}{\sigma}}\Big] \\
\mathcal{D}^{C,D}_s &\equiv 0 \\
\mathcal{D}^{C,C}_s &\equiv (f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}} \\
\mathcal{D}^{C,DC}_s &\equiv \Big[(f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}-(f^D_s-f^C_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\big(\frac{1}{(\tilde{R}^C_s)^{\sigma-1}-(\tilde{R}^D_s)^{\sigma-1}}\big)^{\frac{\sigma-\xi}{\sigma}}\Big] \\
\mathcal{D}^{C,CD}_s &\equiv \Big[(f^C_s-f^D_s)^{\frac{\sigma-\xi}{\sigma}}(\tilde{R}^D_s\tilde{R}^C_s)^{\frac{(\sigma-1)(\sigma-\xi)}{\sigma}}\big(\frac{1}{(\tilde{R}^D_s)^{\sigma-1}-(\tilde{R}^C_s)^{\sigma-1}}\big)^{\frac{\sigma-\xi}{\sigma}}\Big].
\end{aligned}$$

5 Supplementary Material for Quantitative Analysis

5.1 Augmentation

DAC Grants. As another source of financing, I incorporate the DAC grants. In practice, the DAC grants constitute a significant portion of DF (1.3 million counts) along with the DAC (31,459 counts) and Chinese loans (4,400 counts). The median size of the DAC grants ($53,469) are much smaller than those of the DAC loans ($18.7 million) and Chinese loans ($67million).[5] Since the scale of the DAC grant projects are incomparably small to the loan projects while the count is much higher, I model in such a way that they corresponds to the projects near the bottom of productivity distribution and such a way that the augmentation does not qualitatively affect the main results regarding the loans in previous sections. In reality, the DAC grants are also secured after some negotiation process between the applicant country and the DAC agencies. For tractability, I assume that the DAC evaluates the marginal product of each project and equates it to a shadow cost, which represents the cost the borrower would incur if it were a loan contract. Grants are subject to the same monitoring intensity ψ_s^D as DAC loans. Consequently, the optimal size of a grant-financed project j, evaluated by the DAC, $\bar{g}^{EG}_{s,j,t}$, is determined by the same equation as DAC loans: $mpg^E_{sjt} + 1 - \delta^E_s = \tilde{R}^D_s$. However, there is a limit on project size, and the DAC approves projects only if $\bar{g}^{EG}_{s,j,t} \leq T_s$ for some $T_s > 0$. This reflects the practice of many DAC grant agencies, which set a limit on the amount for each individual call for applications. Additionally, grant-financing incurs a fixed cost denoted by f^G_s. Consequently, the effective profit for the government from a grant-financed project, $\tilde{\pi}^G_{s,j,t}$, is given by:

$$\tilde{\pi}^G_{s,j,t} \equiv \int_0^{\bar{g}^{EG}_{s,j,t}} (mpg^E_{s,j,t} - \tilde{R}^D_s + \frac{R^D_s}{\Psi^D_s}) dg^E_{s,j,t} - f^G_s$$

where Ψ^D_s takes the value of ψ^D_s if $\chi \geq R^D_s$ and 1 otherwise. The zero-profit cutoff, which satisfies $\tilde{\pi}^G_{s,t}(\bar{\theta}^G_{s,t}) = 0$, is obtained as:

$$\bar{\theta}^G_{s,t} = \frac{\left((\sigma_s - 1)\frac{f^G_s}{1+(\sigma_s-1)\frac{R^D_s}{\Psi^D_s \tilde{R}^D_s}}\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t} (G^E_{s,t}\tilde{R}^D_s)^{\frac{\sigma_s - 1}{\sigma_s}}.$$

[5] An example of small size grant project is 'Therapy Equipment for Disability and Rehabilitation Centre' in Vietnam to which Australia committed in 2016 to provide $3,640 in 2011 constant dollar term. An example of loan project in the same country and sector is 'Construction of Hai Phong General Hospital' to which South Korea committed in 2017 to provide $87.3 million in 2011 constant USD.

For later convenience, I define $\tilde{f}_s^G \equiv \frac{f_s^G}{1+(\sigma_s-1)\frac{R_s^D}{\Psi_s^D \tilde{R}_s^D}}$. Additionally, I define an extra productivity cutoff, $\bar{\theta}_{s,t}^T$, that equates the optimal project size to the grant size limit, namely $\bar{g}_{s,t}^{EG} = T_s$.

$$\bar{\theta}_{s,t}^T = \frac{\left((\sigma_s - 1)\frac{T_s \cdot (\tilde{R}_s^D)^{1-\sigma_s}}{\sigma_s - 1}\right)^{\frac{1}{\sigma_s}}}{\gamma\gamma_s Y_t}(G_{s,t}^E \tilde{R}_s^D)^{\frac{\sigma_s-1}{\sigma_s}}.$$

Motivated by the fact that the average size of grant projects is almost ten times smaller than loan projects, I make an additional assumption that in sectors with $T_s < \infty$, the DAC sets the grant size limit T_s such that $\bar{\theta}_{s,t}^T = \min\{\bar{\theta}_{s,t}^D, \bar{\theta}_{s,t}^C\}$. This can be implemented by setting $T_s = \min\{f_s^D \cdot (\tilde{R}_s^D)^{\sigma-1}, f_s^C \cdot (\tilde{R}_s^C)^{\sigma-1}\}$.

This assumption implies that the DAC does not allow borrowing countries to receive grants for projects that are productive enough to generate positive effective profits for the government, even if financed by loans. Suppose that $T_s > \min\{f_s^D \cdot (\tilde{R}_s^D)^{\sigma-1}, f_s^C \cdot (\tilde{R}_s^C)^{\sigma-1}\}$, so that $\bar{\theta}_{s,t}^T > \min\{\bar{\theta}_{s,t}^D, \bar{\theta}_{s,t}^C\}$. In this case, the borrowing country would choose DAC grants for some projects, even though it could make positive profits with DAC or Chinese loans. Considering the cost of providing grants without any expected returns, it is unrealistic that the DAC would allow this to happen.

This assumption also excludes the case where $T_s < \min\{f_s^D \cdot (\tilde{R}_s^D)^{\sigma-1}, f_s^C \cdot (\tilde{R}_s^C)^{\sigma-1}\}$. Therefore, there are no projects in the middle of the productivity distribution that are neither eligible for grants nor profitable with loans, which makes the quantification more tractable. It is also likely that the DAC sets the grant and loan conditions in such a way that it does not leave out projects that are fairly productive in the middle of the distribution while financing only less productive projects at the bottom. As a result, the optimal sectoral financing results in Proposition 2 carry over, except that in each category, projects with productivity $\theta \in [\bar{\theta}_{s,t}^G, \min\{\bar{\theta}_{s,t}^D, \bar{\theta}_{s,t}^C\})$ are now financed by DAC grants in addition to the loan-financed projects. The aggregation result in Proposition 3 can also be extended. See Appendix 3.10 for details.

Moreover, allowing for grant-financing potentially gives rise to two additional categories where an entire sector is financed solely by DAC grants, with or without diversion. This is possible when $T_s \to \infty$. I denote each category by $S^{\tilde{G}}$ and S^G.

Self-financing. I also allow for self-financing, where the government does not rely on external sources to finance a project. This is because DF is not available in military sector which constitutes a non-trivial portion of public sector. Generally, if DF is available, self-financing is dominated by DF due to the higher fixed costs associated with other financing sources and will not be commonly used. As a result, self-financing is only considered for sectors where DF is not available.

5.2 Sector Classifcation

Table 15: Sector Classification

Sector name	OECD DAC-5	IMF COFOG
Agriculture, Forestry, Fishing	Agriculture, Forestry, Fishing	Agriculture, Forestry, Fishing, and Hunting
Industry, Mining, Construction	Industry, Mining, Construction	Mining, Manufacturing, Construction
Transport & Storage	Transport & Storage	Transport
Energy	Energy	Fuel and Energy
Communications	Communications	Communication
Health	Health	Health
Education	Education	Education
General Environment Protection	General Environment Protection	Environmental Protection
Water Supply & Sanitation	Water Supply & Sanitation	Housing and Community Amenities
Government & Civil Society	Government & Civil Society; Disaster Prevention & Preparedness	Public Order & Safety
General Budget Support	General Budget Support; Other Multisector	General Public Service; Other Industries
General Economic, Commercial, Labor Affairs	Banking & Financial Services; Business & Other Services; Other Commodity Assistance ; Trade Policies & Regulations	General Economic, Commercial, Labor Affairs; Economic Affairs n.e.c.; Economic affairs R&D
Other Social Infrastructure & Services	Other Social Infrastructure & Services; Population Policies/Programs & Reproductive Health; Development Food Assistance	Recreation Culture Religion; Social Protection
Defense		Defense
	Action Relating to Debt; Emergency Response; Reconstruction Relief & Rehabilitation; Administrative Costs of Donors; Refugees in Donor Countries; Unallocated / Unspecified	

5.3 Estimating Public Capital Sector Shares γ_s

The model predicts that if an advanced country self-finances a development project j in sector s without diversion, the optimal project size would be determined by the following first-order condition:

$$mpg^E_{s,j,t+1} + 1 - \delta_G = \frac{\tilde{U}'_C(C_t)}{\beta \tilde{U}'_C(C_{t+1})}$$

In steady state, the optimal project size is given by:

$$g^{E*}_{s,j} = \Big(\frac{\theta_j \gamma \gamma_s}{1/\beta - (1-\delta_G)}\Big)^\sigma (Y^*)^\sigma (G^{E*}_s)^{1-\sigma}$$

Then, the total expenditure on sector s observed in the data, denoted by G^{O*}_s, is obtained as:

$$\begin{aligned} G^{O*}_s &= \int g^{E*}_{s,j} dj \\ &= \frac{\gamma \gamma_s}{1/\beta - (1-\delta_G)} Y^* \end{aligned}$$

Since data on public capital at the sectoral level is not available, while the IMF COFOG provides public expenditure on each sector each year, I target the investment ratios rather than public capital ratios. In the steady state without diversion, total investment in sector s is simply $I^{G*}_s = \delta_G G^{O*}_s$. Therefore, the ratio of I^{G*}_s to GDP in the steady state is characterized as:

$$\frac{I^{G*}_s}{Y^*} = \frac{\delta^E_s \gamma \gamma_s}{1/\beta - (1-\delta^E_s)}$$

It follows that the share of each sector in total public expenditure is γ_s. I estimate γ_s using Sequential Least Squares Programming (SLSQP), which minimizes the squared distance between γ_s and the mean of the corresponding sector share, with the constraint that $\sum_{s\in\mathcal{S}} \gamma_s = 1$. This approach is equivalent to the Generalized Method of Moments (GMM) with the following moment conditions:

$$\mathbb{E}\Bigg[\gamma_s - \frac{I^O_{r,s,t}}{\sum_{s\in\mathcal{S}} I^O_{r,s,t}}\Bigg] = 0 \quad \text{for each } s \in \mathcal{S}$$

Table 16: Sectoral Public Capital Share

Sector name	**Sector share** γ_s
Agriculture, Forestry, Fishing	0.0119
Industry, Mining, Construction	0.0029
Transport & Storage	0.0573
Energy	0.0053
Communications	0.0004
Health	0.1429
Education	0.1297
General Environment Protection	0.0169
Water Supply & Sanitation	0.0196
Government & Civil Society	0.0449
General Budget Support	0.1434
General Economic, Commercial, Labor Affairs	0.0253
Other Social Infrastructure & Services	0.3613
Defense	0.0382
Sum	**1**

5.4 Estimating Chinese DF Monitoring Intensities ψ_s^C

For the quantitative analysis, I focus on the relative monitoring intensities between DAC and Chinese DF, normalizing the monitoring intensities for DAC DF in all sectors to 1 ($\psi_s^D = 1$). There are two reasons for this approach. First, in the empirical analysis, DAC project sizes are not qualitatively correlated with corruption in most sectors. While I find a correlation in sectors that are difficult to monitor, it is much smaller than the correlation observed for Chinese DF. Secondly, it is extremely challenging to estimate the exact values of monitoring intensities for both DAC and Chinese DF across all sectors since there is no cardinal corruption measure that corresponds empirically to the model's corruption parameter, χ_r. However, under certain identifying assumptions, I can estimate the relative monitoring intensity between DAC and Chinese DF for each sector. To estimate monitoring intensities for Chinese DF, I begin with the model equation that determines the optimal size of effective public capital for project j, $g^E_{r,p,s,j,t}$. The actual size of project j observed in the data, $g^O_{r,p,s,j,t}$, is equal to $g^E_{r,p,s,j,t}/\Psi^p_{r,s}$, where $\Psi^p_{r,s}$ is ψ^p_s if country r diverts DF from provider p in sector s, and 1 otherwise. Hence,

$$g^O_{r,p,s,j,t} = \frac{1}{\Psi^p_s}\left(\frac{\gamma\gamma_s\theta_j}{\tilde{R}^p_{r,s}}\right)^{\sigma} Y^{\sigma}_{r,t}(G^E_{r,s,t})^{1-\sigma}.$$

Taking the log and approximating $\ln \tilde{R}^p_{r,s} = \ln(\frac{R^p_s-(1-\psi^p_s)\chi_r}{\psi^p_s} - (1-\delta_G))$ to the first order around $\chi_r = R^p_s$ and $\psi^p_s = 1$, I obtain:

$$\ln g^O_{r,p,s,j,t} \approx -\ln \Psi^p_s + \sigma \ln \theta_j + \sigma \ln \gamma\gamma_s + \sigma \ln Y_{r,t} + (1-\sigma)\ln G^E_{r,s,t} - \sigma \ln(R^p_s - (1-\delta_G)).$$

Note that the equality holds for $p = D$ since $\psi^D_s = 1$. Since $Y_{r,t}$, $G^E_{r,s,t}$, and $\gamma\gamma_s$ are invariant to p, the difference in the log project size between DAC and Chinese DF arises from three components: monitoring intensity, interest rate, and potential selection bias in productivity θ_j. My model predicts that the productivity cutoffs determining the average size of DAC and Chinese DF projects are driven by the borrowing country's corruption, recipient-provider bilateral and sector-specific fixed costs, and interest rates. Based on this, I control for variables that might affect these factors to account for the systemic difference in the productivity of DAC and Chinese projects. Then, with some additional identifying assumptions, the difference in average project size—controlling for all these factors—can be attributed to the difference in monitoring intensity. Consider the following fixed effect regression model. $\mathbf{X}_{r,p,t}$ includes the gravity variables, bilateral political distance, and $\ln(R^p_s - (1-\delta_G))$.

$$\ln g^O_{r,p,s,j,t} = constant + FE_{s,p} + FE_{r,t} + \mathbf{X}_{r,p,t} \cdot \beta + \epsilon_j$$

I make the following assumptions, where *controls* indicate all the right-hand side variables of the fixed effect model.

- Assumption 1: $\mathbb{P}(\chi_r \geq R_s^C | s, p = C) = 1$
- Assumption 2: $\mathbb{E}\big[\ln \theta_j | p, s, controls\big] = \alpha_{rt} + \alpha_s + \mathbf{X}_{r,p,t}$

Assumption 1 states that all countries using Chinese DF during the sample period are corrupt enough to divert the funds. Considering that the majority of Chinese DF is directed toward countries with higher-than-average corruption indices (Malik et al., 2021), this assumption is reasonable. If anything, the bias would lean toward overestimating the monitoring intensity of Chinese DF. Therefore, if there are recipient countries with insufficient corruption in the sample, the actual monitoring intensity should be lower. As a result, the estimate under this assumption should be considered an upper bound of Chinese DF monitoring intensities relative to the DAC.

The second assumption states that I can control for the difference in average productivity between DAC and Chinese DF in a sector by including recipient-time fixed effects, sector fixed effects, and control variables. Under the two assumptions, the expected values of log project size for DAC and Chinese DF in sector s, given control variables, are:

$$
\begin{aligned}
\mathbb{E}\big[\ln g^O_{r,p,s,j,t} | p = D, s, controls\big] \approx& \sigma \ln \gamma\gamma_s - \sigma \ln(R_s^D - (1 - \delta_G)) \\
&+ \sigma \ln Y_{r,t} + \mathbb{E}\big[(1-\sigma) \ln G^E_{r,s,t} | s, controls\big] \\
&+ \sigma \mathbb{E}\big[\ln \theta_j | p = D, s, controls\big] \\
=& \sigma \ln \gamma\gamma_s - \sigma \ln(R_s^D - (1 - \delta_G)) \\
&+ \sigma \ln Y_{r,t} + \mathbb{E}\big[(1-\sigma) \ln G^E_{r,s,t} | s, controls\big] \\
&+ \alpha_{rt} + \alpha_s + \mathbf{X}_{r,p=D,t} \cdot \beta \\
\mathbb{E}\big[\ln g^O_{r,p,s,j,t} | p = C, s, controls\big] \approx& \sigma \ln \gamma\gamma_s - \sigma \ln(R_s^C - (1 - \delta_G)) \\
&+ \sigma \ln Y_{r,t} + \mathbb{E}\big[(1-\sigma) \ln G^E_{r,s,t} | s, controls\big] \\
&+ \sigma \mathbb{E}\big[\ln \theta_j | p = C, s, controls\big] \\
&- \ln \psi_s^C \cdot \mathbb{P}(\chi_r \geq R_s^C | s, p = C) \\
=& \sigma \ln \gamma\gamma_s - \sigma \ln(R_s^C - (1 - \delta_G)) \\
&+ \sigma \ln Y_{r,t} + \mathbb{E}\big[(1-\sigma) \ln G^E_{r,s,t} | s, controls\big] \\
&+ \alpha_{rt} + \alpha_s + \mathbf{X}_{r,p=D,t} \cdot \beta \\
&- \ln \psi_s^C
\end{aligned}
$$

Then, the difference in sector-provider fixed effects for each sector in the fixed effect regression model is:

$$
\begin{aligned}
FE_{s,p=C} - FE_{s,p=D} =& \mathbb{E}[\ln g^{O}_{r,p,s,j,t} | s, p = C, controls] - \mathbf{X}_{r,p=C,t} \cdot \beta \\
& - (\mathbb{E}[\ln g^{O}_{r,p,s,j,t} | s, p = D, controls] - \mathbf{X}_{r,p=D,t} \cdot \beta) \\
=& - \ln \psi^{C}_{s}
\end{aligned}
$$

Therefore,

$$
\psi^{C}_{s} \approx \exp^{FE_{s,p=D} - FE_{s,p=C}} .
$$

Based on this, I run the fixed effect regressions and use the estimated sector-provider fixed effects for each sector to estimate Chinese DF monitoring intensities. Note that I include only loan projects and exclude grant projects, as grant projects are systematically smaller than loan projects, reflecting differences in productivity that are not fully controlled for by the control variables.

5.5 Estimating Project Productivity Distribution ξ_r

I normalize the Pareto scale parameter, $\underline{\theta}_r$, to 1, as this normalization is innocuous for the quantitative results. I estimate the Pareto shape parameter, ξ_r, for each country (r) using the Maximum Likelihood Estimation (MLE) method, exploiting the properties of the mixture of Pareto distributions. In my model, the pool of potential projects is fixed over time, and the government operates all projects with productivity above a certain cutoff in each period. However, in practice, there may be lags between the government's planning and the actual implementation of each project. These delays could be due to various factors, such as lengthy negotiations with DF providers or domestic administrative or legislative lags, which are beyond the scope of this paper.

As a result, in the data, each project appears with some randomness in different years. Moreover, only the information on the initial commitment is fully observable in the project-level data, and each project does not reappear in later years. In other words, projects are sporadically observed in different years regardless of their productivity. To calibrate the distribution of a fixed project pool to the data, I pool all the projects in a way that leverages the unique properties of the mixture of Pareto distributions. It turns out that I can estimate the shape parameter, ξ_r, by simply pooling all the observations.

Suppose there are k distributions with respective probability density functions $f_1(x)$, $f_2(x)$, ..., $f_k(x)$, with supports $\mathbb{S}_1$, $\mathbb{S}_2$, ..., $\mathbb{S}_k$, and positive mixing probabilities p_1, p_2, ..., p_k, where $\sum p_i = 1$. It is well known that a random variable X from the mixture distribution has a pdf $f(x) = \sum_{i=1}^{k} p_i f_i(x)$, with support $x \in \cup_i \mathbb{S}_i$ (Hogg et al., 2013).

Recall that the size of project i financed by provider p observed in year t is determined by the following equation:

$$g^O_{r,p,s,j,t} = \frac{1}{\Psi^p_s}\left(\frac{\theta_j \gamma \gamma_s}{\tilde{R}^p_s}\right)^{\sigma} Y_t^{\sigma}(G^E_{s,t})^{1-\sigma}.$$

If θ_j follows a Pareto distribution with shape parameter ξ_r and scale parameter $\underline{\theta}_r$, then the distribution of project sizes financed by p in year t in sector s also follows a Pareto distribution but with shape parameter $\frac{\xi_r}{\sigma}$ and scale parameter $\underline{\theta}_{r,s,p,t} \equiv \frac{1}{\Psi^p_s}\left(\frac{\gamma\gamma_s}{\tilde{R}^p_s}\right)^{\sigma} Y_t^{\sigma}(G^E_{s,t})^{1-\sigma}\underline{\theta}^{\sigma}_{r,s}$. Let $f_{r,s,p,t}(x; \frac{\xi_r}{\sigma}, \underline{\theta}_{r,s,p,t})$ denote the corresponding pdf for all p and t. Also, let $N_{r,s,p,t}$ denote the number of projects observed in year t for provider p in sector s, and define $w_{r,s,p,t} \equiv N_{r,s,p,t}/\sum_{p,t} N_{r,s,p,t}$. Then, the pdf of project size from the pooled sample can be written as:

$$f_r(x) = \sum_{p,t,s} w_{r,s,p,t} \cdot f_{r,s,p,t}(x; \frac{\xi_r}{\sigma}, \underline{\theta}_{r,s,p,t})$$

Note that all $f_{r,s,p,t}$ share the same shape parameter $\frac{\xi_r}{\sigma}$. As a result, the closed-form expression for f_r is:

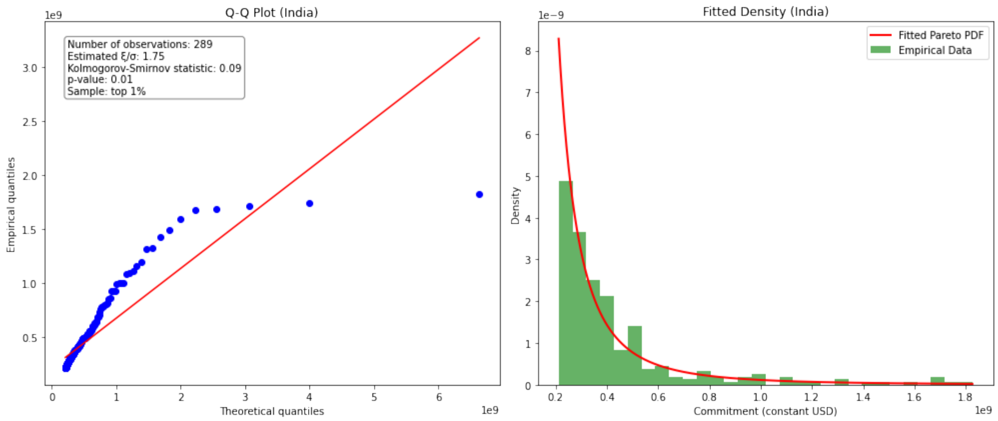
$$f_r(x) = \frac{\frac{\xi_r}{\sigma}\left(\left[\sum_{p,s,t} w_{r,s,p,t} \cdot \underline{\theta}_{r,s,p,t}\right]^{\frac{\sigma}{\xi_r}}\right)^{\frac{\xi_r}{\sigma}}}{x^{\frac{\xi_r}{\sigma}+1}},$$

which is in the same form as a Pareto distribution with shape parameter $\frac{\xi_r}{\sigma}$ and scale parameter $\tilde{\theta}_r \equiv \left[\sum_{p,s,t} w_{r,s,p,t} \cdot \underline{\theta}_{r,s,p,t}\right]^{\frac{\sigma}{\xi_r}}$. Based on this result, I fit the right tail of the pooled sample using the Pareto distribution and estimate $\frac{\xi_r}{\sigma}$. In doing so, I maximize the following log-likelihood function:

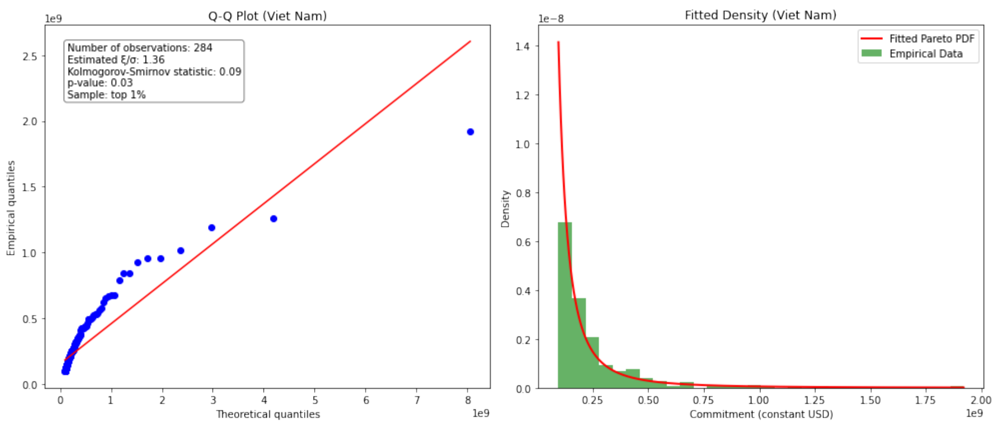
$$log\mathcal{L}(\frac{\xi_r}{\sigma}, \tilde{\theta}_r) = \sum_{i=1}^{N_r} log f_r(x_i; \frac{\xi_r}{\sigma}, \tilde{\theta}_r).$$

I focus on fitting the right tail rather than using all observations, following the literature that utilizes the Pareto distribution. In the firm dynamics and trade literature studying the distribution of firm sizes, the Pareto distribution is widely adopted not only for its analytical convenience but also for its ability to approximate the right tail of the distribution (Arkolakis et al., 2012). Similarly, the assumption of a Pareto distribution provides analytical convenience for aggregation in my model and empirically explains the right tail of the distribution of public project sizes. However, it is well known that the Pareto distribution may not provide a good fit for the entire distribution. More importantly, when estimating the shape parameter using the full sample, the estimated value often fails to meet theoretical requirements (Head et al., 2014).

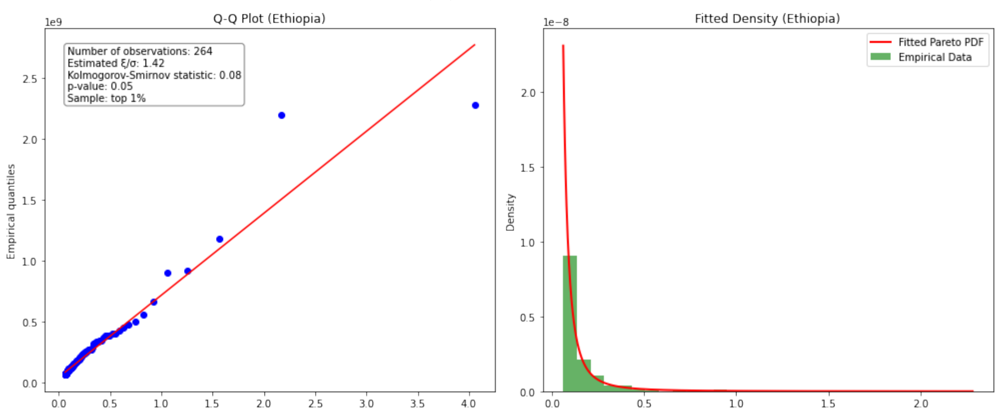
My model faces the same issue, as it requires $\xi_r > \sigma$ and that the estimated value for $\xi_r > \sigma$ be greater than 1. Therefore, I take a similar approach to Head et al. (2014) by fitting the right tail of the distribution. For each recipient, I fit the top 1 percent of samples and estimate the shape parameter. Among 112 countries with enough sample sizes (> 30), all except for 17 have estimates of ξ_r/σ greater than 1. For those with estimates lower than 1 and those with less than 30 projects at the top 1%, I set the value to 1.014, which is the lowest estimate among those greater than 1. Figure 5 shows the histogram of estimated ξ_r/σ. Figure 4 shows the QQ plot and fitted density of the projects with summary statistics for three selected countries with the most sample size.



(a) Kenya



(b) Vietnam



(c) Ethiopia

Figure 4: QQ Plot and Fitted Density of Selected Economies

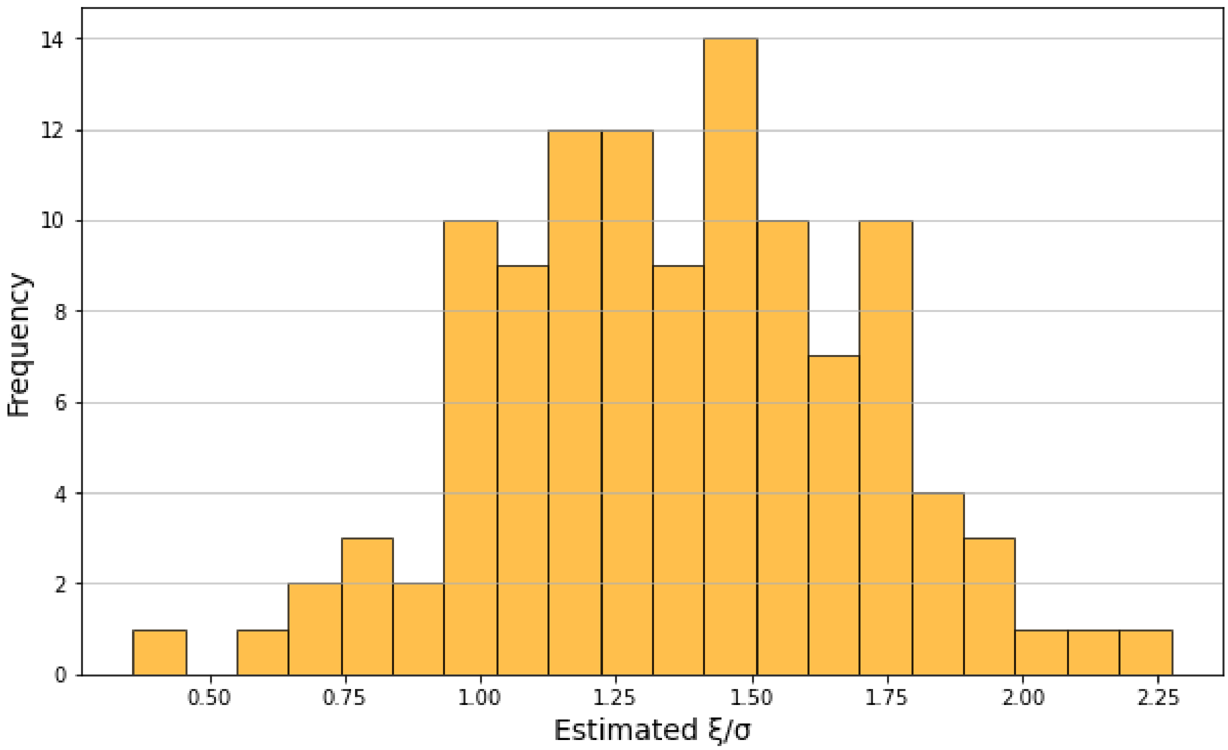


Figure 5: Histogram of Estimated ξ/σ

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